MORREY'S REPRESENTATION THEOREM FOR SURFACES IN METRIC SPACES

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1. Introduction. In 1935 Morrey showed that a non-degenerate surface of finite Lebesgue area has a quasi-conformal representation on the unit circle. He made use of Schwarz' result for polyhedral surfaces and was able to use a limiting process after he had shown that the representations of the surfaces involved were sufficiently well behaved for the area to be given by the usual integral. The limiting process depended upon Tonelli's result concerning the lower semi-continuity of the Dirichlet integral.

Several years later Cesari reduced the dependence upon complex variable theory by the use of a variational technique to obtain a slightly weaker version of Schwarz' result, but he showed that for the remainder of Morrey's argument his form was adequate.

The purpose of this paper is to remove the restriction that the surfaces be in Euclidean space; the method is that of Cesari.

Morrey's theorem has proved useful in the study of certain two-dimensional problems in the calculus of variations. It is hoped that the extension of his theorem will permit corresponding extensions of that theory [3, 6, 12].

A desirable feature of quasi-conformal mappings is that the area of the surface is given by one half the Dirichlet integral. To retain this property for surfaces which are not in Euclidean space requires the definition of an appropriate integral to complement the definition of area. The definition of (Lebesgue) area used in this paper is that given in [13] which agrees with the usual definition in case the surface is in Euclidean space.

We shall make use of the ideas of [13] in two other respects. First, we need only solve our problem for surfaces in m, the space of bounded sequences [1], since the definitions are chosen so as to be invariant under an isometry and we can map other surfaces isometrically into m. Second, we shall make use of the fact that the area of a function in m depends only upon its distinct components. The last remark results from the definition of the area of a triangle. Let $r=\{r^i\}$, $s=\{s^i\}$, and $t=\{t^i\}$ be three points in m. Then the area of the triangle with these points as vertices is, by definition,

$$\frac{1}{2}\sup_{i,k}\begin{vmatrix} r^i & r^k & 1 \\ s^i & s^k & 1 \\ t^i & t^k & 1 \end{vmatrix}.$$

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