ON THE GENERALIZED RADIATION PROBLEM OF A. WEINSTEIN

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1. Introduction. The generalized radiation problem as formulated and solved by A. Weinstein [8] requires determination of a non-singular solution of the two-dimensional Euler-Poisson-Darboux (abbreviated EPD) equation

$$u_{xx}^{[k]}=u_{yy}^{[k]}+\frac{k}{y}u_y^{[k]} \tag{1.1}$$

for $-\infty<k<1$ such that

$$\lim_{y\to 0} u^{[k]}(x, y)=f(x) \quad \text{and} \quad u^{[k]}(x, y)=0 \quad \text{for} \quad y=x \tag{1.2}$$

where $f(x)$ is a function given on some interval $0\leqq x\leqq a$, possessing a specified number of continuous derivatives there and having another specified number of zero derivatives at $x=0$. These conditions on $f(x)$ depend on the parameter k as stated in [8]. The classical radiation problem, requiring an axially symmetric solution of the higher dimensional wave equation with a certain type of singularity, as given in [3], is a special case. If k is an integer and $u^{[k]}$ a solution of the above generalized radiation problem, then

$$u^{(2-k)}(x, y)=\frac{u^{[k]}(x, y)}{y^{1-k}} \tag{1.3}$$

is a solution of the classical radiation problem in an $m=3-k$ dimensional space (not counting time as a dimension). Thus from a regular solution $u^{[k]}$ one generates a solution $u^{[2-k]}$ of the EPD equation with that type of singularity needed to solve the radiation problem.

The first part of this paper will be devoted to uniqueness for the generalized radiation problem. Although a more complete answer to the uniqueness question would be welcome, consideration of solutions which have two continuous derivatives on $y=x$ is natural since such solutions are the ones that correspond closely to radiation phenomena. Let T be a triangle with vertices $(0, 0)$, $(a, 0)$, $(a/2, a/2)$. We define a function to be regular on T if it has two continuous derivatives in some triangle G the interior of which contains T and its sides except for the base line, $y=0$. Only a function satisfying the EPD equation, regular on T, and taking on the given data will be considered a solution of the

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