T-SETS AND ABSTRACT (L)-SPACES

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1. Introduction. The theory of T-sets and of F_T-functionals was developed [4] in reference to abstract (M)-spaces for application to the characterization of Banach spaces which may be represented as Banach spaces of continuous functions. The purpose of this paper is to discuss their use in reference to abstract (L)-spaces [3] for application to the representation of certain Banach spaces as spaces of integrable functions.

A distinction of three types of abstract (L)-spaces is first made and illustrated. Next an extremely simple characterization of the Banach spaces which are susceptible of a semi-ordering under which they become abstract (L)-spaces of the second or third type is established. Then a complete analysis of the role of T-sets and of F_T-functionals in the third and most important type of abstract (L)-space is given. Finally a few remarks are appended relative to T-sets in abstract (L)-spaces of the first type.

2. Preliminary concepts. Let BL be a semi-ordered Banach space which is a linear lattice under its semi-ordering, and in which the collection P of elements $a \geqq 0$ is closed with respect to the norm. Consider, with reference to the subset P of BL, three possible additional requirements:

(I) If $a,\ b \in P$, then $\|a+b\| = \|a\| + \|b\|$.

(II) If $a,\ b \in P$, then $\|a+b\| = \|a\| + \|b\|$, and P is a subset of BL maximal with respect to this property.

(III) If $a,\ b \in P$, then $\|a+b\| = \|a\| + \|b\|$, and if $a \wedge b = 0$, then $\|a-b\| = \|a+b\|$.

A space BL wherein the subset P possesses property III is usually called an abstract (L)-space. If property III obtains in P, then property II also obtains in P with respect to BL. Thus for any $a \in BL$ with $a \notin P$, $a = a^+ - a^-$ with $a^+,\ a^- \in P$, $a^+ \wedge a^- = 0$, while $a^- \neq 0$. Then

$$\begin{aligned}\|a+a^-\| &= \|a^+\| < \|a^+\| + \|a^-\| + \|a^-\| \\ &= \|a^+ + a^-\| + \|a^-\| = \|a^+ - a^-\| + \|a^-\| = \|a\| + \|a^-\|,\end{aligned}$$

so that P is maximal in BL with respect to the stated property. Thus for the subset P of BL, we have III $\Rightarrow$ II $\Rightarrow$ I. It will presently be seen, however, that I does not imply II and that II does not imply III. Hence let BLI denote the space BL under the additional assumption

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