THE TAUBERIAN THEOREM FOR GROUP ALGEBRAS OF VECTOR-VALUED FUNCTIONS

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1. Introduction. The object of this paper is to prove the ideal-theoretic version of Wiener's tauberian theorem for algebras which we will call *group algebras of vector-valued functions*. These algebras are defined as follows. Let $G=\{a, b, \cdots\}$ denote a locally compact abelian group and let $X=\{x, y, \cdots\}$ represent a complex commutative Banach algebra. Our group algebra $B=B(G, X)$ consists of the set of all measurable absolutely integrable functions defined over G with values in X. Of course we must identify functions which differ on sets of Haar measure 0. As norm for an element $f \in B$ we take

$$\|f\|_B=\int_G |f(a)|_X \, da \ .$$

(Hereafter, we will omit an indication of the domain of integration if the integral is taken over the entire group G.) The space $B(G, X)$ is known to be complete in the given norm [4]. Further, we introduce into B the following operations

$$(f+g)(a)=f(a)+g(a) \ , \qquad (\lambda f)(a)=\lambda f(a)$$

where λ is a complex number, and

$$(f*g)(a)=\int f(b)g(a-b) \, db$$

where the integral is taken in the sense of Bochner [1, 4] with respect to Haar measure db. The algebra $B(G, X)$ thus becomes, as is easily shown, a complex commutative Banach algebra which specializes into the classical group algebra $L(G)$ if X is chosen as the complex numbers. It is these algebras $B(G, X)$ which will be the object of our study.

The tauberian theorem for $B(G, X)$ will be proved by appealing to a theorem in the general theory of Banach algebras (see [5], p. 85 corollary, or [6], Theorem 38.) This latter result might be designated as the "general tauberian theorem." It says that if a complex commutative B-algebra Y is semi-simple, regular, and is such that the set of $y \in Y$ with $\phi_M(y)$ having compact support in $\mathfrak{M}(Y)$ is dense in Y, then every proper closed ideal in Y is contained in a regular maximal ideal.

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