(γ, k)-SUMMABILITY OF SERIES

U. C. GUHA

1. Introduction. Let $\gamma_k(x)$ denote the (C, k) mean of $\cos x$, so that

(1.1) $$\gamma_0(x)=\cos x \ ,$$

and

(1.2) $$\gamma_k(x)=\frac{k}{x^k}\int_0^x (x-u)^{k-1}\cos u\,du \ , \quad (k>0),$$
$$=k\int_0^1 (1-t)^{k-1}\cos xt\,dt \ ,$$
$$=\Gamma(k+1)\frac{C_k(x)}{x^k} \ ,$$

where $C_k(x)$, the kth fractional integral of $\cos x$, is commonly known as Young's function [**6**, p. 564].

We shall say that the infinite series $\sum_0^\infty a_n$ is summable (γ, k) if

(i) $\sum_0^\infty a_n\gamma_k(nt)$ converges for $0<t<A$

and

(ii) $\lim_{t\to 0}\sum_0^\infty a_n\gamma_k(nt)=S$, where S is finite.

We see that $(\gamma, 1)\equiv(R, 1)$ and $(\gamma, 2)\equiv(R, 2)$, where $(R, 1)$ and $(R, 2)$ are the well known Riemann summability methods. Hence the (γ, k)-summability methods constitute, in a sense, an extension of $(R, 1)$ and $(R, 2)$ summability methods to (R, k) methods where k may be non-integral. But this extension is not linked with the ideas which lie at the root of the Riemann summability methods, that is, taking generalised symmetric derivatives of repeatedly integrated Fourier series, so that the equivalence of (γ, k) and (R, k) for $k=1, 2$ may be considered to be somewhat accidental, and the extension artificial. However, the (γ, k) methods are also connected with certain aspects of the summability problems of Fourier series. For, let $\sum_0^\infty A_n(x)$ be the Fourier series of a periodic and Lebesgue integrable function $f(x)$ and let

$$\phi(t)=\frac{1}{2}\{f(x+t)+f(x-t)\} \ .$$

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