ON TWO THEOREMS OF PHRAGMÉN-LINDELÖF FOR LINEAR ELLIPTIC AND PARABOLIC DIFFERENTIAL EQUATIONS OF THE SECOND ORDER

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1. Introduction. In Part I of this paper our main interest is to generalize to elliptic equations the following theorem of Phragmén-Lindelöf:

THEOREM 0. *If* $f(z) \to a$ *as* $z \to \infty$ *along two straight lines, and* $f(z)$ *is regular and bounded in the angle between them, then* $f(z) \to a$ *uniformly in the whole angle as* $z \to \infty$.

A generalization of the classic Phragmén-Lindelöf theorem to elliptic equations was given by Gilbarg [1] and Hopf [4]. A refined form of that classic theorem, due to the Nevanlinnas [5], [6; 42–44] and Heins [3], was generalized to elliptic equations by Serrin [8].

In generalizing Theorem 0 we shall make an extensive use of the Gilbarg-Hopf results.

In Part II we generalize to parabolic equations both the classic Phragmén-Lindelöf Theorem and Theorem 0.

In § 2, Theorem 0 is proved for elliptic equations defined in any 2-dimensional domains (Theorems 1, 2). The case $n>2$ is treated in § 3, for domains contained in a half space. In § 4 we consider the behavior of solutions in an angular neighborhood of the origin, and we obtain results similar to those of §§ 2, 3. In §§ 5, 6, generalizations to parabolic equations are given: Theorems 7, 9 extend the classic Phragmén-Lindelöf Theorem and Theorems 8, 10 extend Theorem 0.

The results in Part I are somewhat analogous with Theorems 2, 3, 3′ of Gilbarg-Serrin's paper [2]. The similarity appears both in the type of conditions imposed on the coefficients of the elliptic operator and in the assertions. It is however important to note that our results cannot be obtained by the Gilbarg-Serrin methods, since Harnack Inequalities which play an essential role in their paper, do not hold uniformly in open domains.

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