CHARACTERISTIC DIRECTION FOR EQUATIONS OF MOTION OF NON-NEWTONIAN FLUIDS

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1. Introduction. According to the Reiner-Rivlin theory of non-Newtonian fluids,[1] the stress tensor t^i_j is given in terms of the rate of strain tensor d^i_j by relations of the form

$$t^i_j = -p\delta^i_j + \mathscr{F}_1 d^i_j + \mathscr{F}_2 d^i_k d^k_j \,, \tag{1}$$

where p is an arbitrary hydrostatic pressure, the $\mathscr{F}$'s are essentially arbitrary differentiable functions of

$$\mathrm{II} = -\frac{1}{2} d^i_j d^j_i \,, \qquad \mathrm{III} = \det d^i_j \,, \tag{2}$$

and d^i_j satisfies the incompressibility condition

$$d^i_i = 0 \,. \tag{3}$$

The tensors d^i_j and t^i_j are both symmetric.

It is known [2] that the characteristic directions of the corresponding equations of motion are the unit vectors ν_i satisfying

$$F(\nu_i) \equiv 2U^2 + 2UU^i_i + (U^i_i)^2 - U^i_j U^j_i = 0 \,, \tag{4}$$

where

$$\begin{aligned} U &= \mathscr{F}_1 + \mathscr{F}_2 \mu^i \nu_i \,, \\ U^i_j &= \mathscr{F}_2 (d^i_j - \nu^i \mu_j) + 2(\mu^i - \nu^i \mu_k \nu^k)\left(\mu^m d_{mj} \frac{\partial \mathscr{F}_1}{\partial \mathrm{III}} - \mu_j \frac{\partial \mathscr{F}_1}{\partial \mathrm{II}}\right) \\ &\quad + 2(d^i_m \mu^m - \nu^i \mu_m \mu^m)\left(\mu^n d_{nj} \frac{\partial \mathscr{F}_2}{\partial \mathrm{III}} - \mu_j \frac{\partial \mathscr{F}_2}{\partial \mathrm{II}}\right) \,, \\ \mu_i &= d_{ij} \nu^j \,. \end{aligned}$$

Since $F(\nu_i)$ is a continuous function of ν_i on the compact set $\nu_i \nu^i = 1$, a necessary and sufficient condition that no real characteristic directions exist is that $F(\nu_i)$ be of one sign for all unit vectors. Using this fact, we obtain simpler necessary conditions which are shown to be sufficient when $\mathscr{F}_2 \equiv 0$.

2. Necessary conditions. Let d_1, d_2 and d_3 denote the eigenvalues of d^i_j. From (3),

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[1] This theory was proposed independently by Reiner [4] for compressible fluids, by Rivlin [5] for incompressible materials. We treat the latter case.