PRINCIPAL SOLUTIONS OF NON-OSCILLATORY SELF-ADJOINT LINEAR DIFFERENTIAL SYSTEMS

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1. Introduction. In their study of real quadratic functionals

$$\int_a^b [r(x)y'^2+2q(x)yy'+p(x)y^2]dx$$

admitting a singularity at the end-point $x=a$ Morse and Leighton [11] showed that if $x=a$ is not its own first conjugate point then the corresponding Euler differential equation

$$(1.1)\qquad (r(x)y'+q(x)y)'-(q(x)y'+p(x)y)=0,\quad a<x\leqq b\ ,$$

possesses a non-trivial solution $u(x)$ such that $u(x)/y(x)\to 0$ as $x\to a^+$ for each solution $y(x)$ of (1.1) that is independent of $u(x)$. Such a solution $u(x)$ was termed a focal solution belonging to $x=a$ by Morse and Leighton [11], but in a subsequent continuation of the study by Leighton [8] the terminology was changed to principal solution.

If $f(t)$ is a real-valued continuous function on $t_0\leqq t<\infty$ and

$$(1.2)\qquad x''+f(t)x=0\ ,\qquad t_0\leqq t<\infty\ ,$$

is non-oscillatory, Hartman and Wintner [4] have termed a non-trivial solution $x(t)$ a principal solution if

$$(1.3)\qquad \int_{t_0}^\infty |x(t)|^{-2}dt=\infty\ ,$$

for t_0 greater than the largest zero of $x(t)$, and proved that a non-oscillatory equation (1.2) has a principal solution that is unique to an arbitrary non-zero constant factor; moreover, if $x(t)\not\equiv 0$ is a solution of (1.2) which is not principal then every solution $y(t)$ of (1.2) is of the form $y(t)=Cx(t)+o(|x(t)|)$ as $t\to\infty$, where the constant C is or is not zero according as $y(t)$ is or is not principal. In view of this latter result, for a non-oscillatory equation (1.2) a solution $x(t)$ is principal in the sense of Hartman and Wintner if and only if it is principal in the sense of Morse and Leighton.

Recently Hartman [5] has considered a self-adjoint vector differential equation

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