ON THE PERIODICITY OF THE SOLUTION OF A CERTAIN NONLINEAR INTEGRAL EQUATION

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In the following paper we will study the nonlinear integral equation

$$E(t)=F(t)-\int_0^t G(t-\tau)N\{E(\tau)\}\,d\tau \tag{1}$$

where $F(t)$ is a known periodic real function and $G(t)$ and $N(x)$ are known real functions. In particular we will investigate the behaviour of the solution $E(t)$ of the equation (1) for large values of t.

We assume that $G\in L[0, \infty]$ and that $N(x)$ is bounded almost everywhere and Borel-measurable in $[-\infty, \infty]$. Furthermore $N(x)$ is assumed expressible in the form

$$N(x)\sim N(0)+\int_{-\infty}^{+\infty}S(\lambda)\frac{e^{i\lambda x}-1}{i\lambda}d\lambda \tag{2}$$

with $\int_{-\infty}^{+\infty}|S(\lambda)|d\lambda<\infty$ and with finite $N(0)$. This representation is to be valid almost everywhere in $[-\infty, \infty]$

Because $N(x)$ is Borel-measurable in $[-\infty, \infty]$ and $|N(0)|<\infty$, the measurability of x implies the measurability of $N(x)$. The following four classes of $N(x)$-functions are distinguished:

$$\begin{array}{llll} N\in K_{11} & \text{if} & x\in L[0,\ 1] & \text{implies } N(x)\in L[0,\ 1] \\ N\in K_{1\infty} & \text{if} & x\in L[0,\ 1] & \text{implies } N(x)\in L[0,\ \infty] \\ N\in K_{\infty 1} & \text{if} & x\in L[0,\ \infty] & \text{implies } N(x)\in L[0,\ 1] \\ N\in K_{\infty\infty} & \text{if} & x\in L[0,\ \infty] & \text{implies } N(x)\in L[0,\ \infty] \end{array} \tag{3}$$

The space of measurable and bounded functions defined on the finite interval $[0, A]$ will be denoted by $M[0, A]$. The norm of $x\in M[0, A]$ is defined, as usual, by

$$||x||=\inf_E\left\{\sup_{t\in[0,A]-E}|x|\right\}$$

where E ranges over the sets of measure zero in $[0, A]$, and the distance of $x\in M[0, A]$ and $y\in M[0, A]$ by $||x-y||$. The space $M[0, 1]$ is complete.

The proofs in this paper will be based on the following theorem by Tihonov (see for instance [1]) which is valid in $M[0, A]$: Let the operator B map $M[0, A]$ into itself and let $||B(x)-B(y)||\leqq\beta||x-y||$ for all x and

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