INVERSION AND REPRESENTATION THEOREMS FOR A GENERALIZED LAPLACE INTEGRAL

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1. Varma [8] introduced a generalization of the Laplace integral

$$(1) \qquad \mathscr{F}(x) = \int_0^\infty e^{-xt}\phi(t)dt$$

in the form

$$(2) \qquad F(x) = \int_0^\infty (xt)^{m-1/2}e^{-xt/2}W_{k,m}(xt)\phi(t)\,dt$$

where $\phi(t) \in L(0, \infty)$, $m > -1/2$ and $x > 0$. This generalization is a slight variant of an equivalent integral introduced earlier by Meijer [7] and reduces to (1) when $k + m = 1/2$. In a recent paper [1] Erdélyi has pointed out that the nucleus of (2) can be expressed as a fractional integral of e^{-xt} in terms of the operators of fractional integration introduced by Kober [6]. In this note two theorems have been given-one giving an inversion formula for the transform (2) and another giving necessary and sufficient conditions for the representation of a function as an intgral of the form (2) by considering its nucleus as a fractional integral of e^{-xt}.

2. The operators are defined as follows.

$$I^+_{\eta,\alpha}\mathscr{F}(x) = \frac{1}{\Gamma(\alpha)}x^{-\eta-\alpha}\int_0^x (x-u)^{\alpha-1}u^\eta \mathscr{F}(u)du$$

$$K^-_{\zeta,\alpha}\mathscr{F}(x) = \frac{1}{\Gamma(\alpha)}x^\zeta\int_x^\infty (u-x)^{\alpha-1}u^{-\zeta-\alpha}\mathscr{F}(u)du$$

where $\mathscr{F}(x) \in L_p(0, \infty)$, $1/p + 1/q = 1$ if $1 < p < \infty$, $1/q = 0$ if $p = 1$, $\alpha > 0$, $\eta > -1/q$, $\zeta > -1/p$.

The Mellin transform $\overline{M}_t\mathscr{F}(x)$ of a function $\mathscr{F}(x) \in L_p(0, \infty)$ is defined as

$$\overline{M}_t\mathscr{F}(x) = \int_0^\infty \mathscr{F}(x)x^{it}dx \qquad (p = 1)$$

and

$$= \underset{X\to\infty}{\overset{\text{index } q}{\text{l.i.m}}}\int_{1/X}^X \mathscr{F}(x)x^{it-1/q}dx \qquad (p > 1)$$

Received May 8, 1958.