DISTAL TRANSFORMATION GROUPS

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Let X be a topological space and G a group of homeomorphisms of X onto itself. Then G is said to be *distal* if given any three points x, y, z in X and any filter $\mathscr{F}$ on G, then $x\mathscr{F} \to z$ and $y\mathscr{F} \to z$ implies that $x = y$. The above definition of distal is a topological variant of the one given in [2]; the two notions coincide when the underlying space X is compact.

This paper deals with two topics in the study of distal transformation groups. First, a recursive characterization of these groups is given in a general setting, and second it is shown that under suitable restrictions on X and G, distal is a property strong enough to imply equicontinuity of G. In order to make this statement precise a few definitions are needed. For a complete discussion of the following notions, the reader is referred to [2].

Let a, b be functions of X into X and let $x \in X$. Then xa will denote the image of x under a, and ab the composite function first a then b. Under the operation of composition X^X is a semigroup such that the maps $b \to ab$ $(b \in X^X)$ are continuous for all $a \in X^X$, and the maps $b \to ba$ $(b \in X^X)$ are continuous for all continuous functions a of X into X. The group G may be regarded as a subset of X^X and its closure T formed. One may also consider S the closure of G in the topology of uniform convergence on X. When X is compact, S is a topological group of homeomorphisms of X onto X but is in general not compact, whereas T is compact but is in general not a group. Hence in studying T instead of S the emphasis is on the algebraic rather than the topological structure.

A subset A of G is said to be *syndetic* if there exists a compact subset K of G such that $AK = G$. (If no topology is specified for G, then it is assumed to be provided with the discrete topology.) A point $x \in X$ is an *almost periodic point with respect to* G if given any neighborhood U of x, there exists a syndetic subset A of G such that $xA = [xa \mid a \in A] \subset U$. If every point of X is an almost periodic point with respect to G, then G is said to be *pointwise almost periodic*.

Let I be a set with cardinal number $\alpha > 0$. Then each $g \in G$ induces a homeomorphism $(x_i \mid i \in I) \to (x_i g \mid i \in I)$ of X^α onto X^α which will also be referred to as g. Under this identification G becomes a group of

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