A SET FUNCTION DEFINED FOR CONVEX PLANE DOMAINS

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Introduction In this note, we define a set function for bounded plane convex domains and study its properties. In particular, we exhibit its analogy to a classical set function from the theory of conformal mapping. The function is of interest because it is involved in bounds and approximations to several physical quantities, notably the torsional rigidity.

1. Preliminary definitions Let D denote a bounded open convex plane domain and let C denote the boundary of D. For each $p \in D$ and each $q \in C$, we denote by h_{pq} the distance (regarded as positive) from p to the supporting line to D at q. The function h_{pq} is single valued for almost all q.

Consider the family of functions defined by the integrals

$$\oint_C h_{pq}^{\alpha} ds_q \,, \tag{1}$$

where ds_q denotes an arc-element of C at q.

For most values of the exponent α, the integral in (1) depends upon p as well as D. For the special cases $\alpha = 0{,}1$ the integral (1) is independent of p. For $\alpha = 0$ we obtain the length of C which we denote by

$$L(D) = L = \oint_C ds_q \,. \tag{2}$$

For $\alpha = 1$, we obtain twice the area of D, which we denote by

$$2A(D) = 2A = \oint_C h_{pq} ds_q \,. \tag{3}$$

In this paper we will be primarily concerned with the value $\alpha = -1$. For this case we employ the notation

$$B_p(D) = B_p = \oint_C h_{pq}^{-1} ds_q \,. \tag{4}$$

If D is transformed by a similarity transformation (i.e. linear isogonal map) into D^* and if p is transformed into p^* then

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