ON THE DETERMINATION OF NUMBERS BY THEIR SUMS OF A FIXED ORDER

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1. Introduction. We wish to treat the following problem (suggested by a problem of L. Moser [2]):

Let $\{x\} = \{x_1, \cdots, x_n\}$ be a set of complex numbers (if one is interested in generality, one may consider them elements of an algebraically closed field of characteristic zero) and let $\{\sigma\} = \{\sigma_1, \cdots, \sigma_{\binom{n}{s}}\}$ be the set of sums of s distinct elements of $\{x\}$. To what extent is $\{x\}$ determined by $\{\sigma\}$ and what sets can be $\{\sigma\}$ sets?

In § 2 we answer this question for $s = 2$. In § 3 we treat the question for general s.

2. The case $s = 2$.

THEOREM 1. *If $n \neq 2^k$ then the first n elementary symmetric functions of $\{\sigma\}$ can be prescribed arbitrarily and they determine $\{x\}$ uniquely.*

Proof. Instead of the elementary symmetric functions we consider the sums of powers, setting

$$\sum\nolimits_k = \sum_{i=1}^{\binom{n}{2}} \sigma_i^k , \qquad S_k = \sum_{i=1}^{n} x_i^k .$$

Then

$$(1) \qquad \sum\nolimits_k = \sum_{i=1}^{\binom{n}{2}} \sigma_i^k = \sum_{1 \le i_1 < i_2 \le n} (x_{i_1} + x_{i_2})^k = \frac{1}{2} \sum_{\substack{i_1, i_2 = 1 \\ i_1 \neq i_2}}^{n} (x_{i_1} + x_{i_2})^k$$

$$= \frac{1}{2}\left(\sum_{i_1, i_2 = 1}^{n} (x_{i_1} + x_{i_2})^k - \sum_{i=1}^{n} (2x_i)^k \right) .$$

Expanding the binomials and collecting like powers we obtain

$$\sum\nolimits_k = \frac{1}{2}\left(\sum_{l=0}^{k} \binom{k}{l} S_l S_{k-l} - 2^k S_k \right)$$

$$= \frac{1}{2}(2n - 2^k) S_k + \frac{1}{2} \sum_{l=1}^{k-1} \binom{k}{l} S_l S_{k-l}$$

Thus, since the coefficient of S_k does not vanish, we can solve re-

Received May 16, 1958.