CENTRALIZERS IN JORDAN ALGEBRAS

BRUNO HARRIS

Introduction. The aim of this paper is to prove for Jordan algebras some theorems on centralizers of subalgebras analogous to known results in the theory of associative algebras (contained in [6, Chapter 3] and [7, Chapter 6], for instance).

The definition of the centralizer of a subalgebra in a Jordan algebra is based on the concept of "operator commutativity" introduced by Jordan, von Neumann and Wigner in [17]: two elements x, y of the Jordan algebra J operator commute if the operators $R_x: a \to ax$ and $R_y: a \to ay$, acting on J, commute, that is $(ax)y = (ay)x$ for all elements a of J. In §1 we study this concept, extend the results of [8] to algebras over fields of characteristic not two, and show that for many types of Jordan algebras obtained from associative algebras by introducing the Jordan product $a \circ b = ab + ba$ (ab the associative product), the centralizer of a subalgebra is just the set of elements commuting in the associative multiplication with the elements of the subalgebra. Thus some of our later results can be regarded as generalizations of the associative algebra results if we convert the associative algebras into Jordan algebras by means of the Jordan product.

In §2 we generalize some of the theory of a single linear transformation in a finite dimensional vector space (see [6, Chapter 3] and [13]) to the subalgebra generated by a single element in a simple finite dimensional Jordan algebra. We show that such a subalgebra is equal to the centralizer of its centralizer, and we also generalize to any central simple Jordan algebra a formula of Frobenius giving the dimensionality of the centralizer of a single linear transformation in terms of the degrees of its invariant factors. A special case of this formula—namely, the formula for the central simple Jordan algebra of all symmetric matrices—was proved earlier, and by a different method, by H. Osborn (to appear in these Transactions).

In §3 we study the centralizer theory of a simple subalgebra in a central simple Jordan algebra. We show that the analogues of the centralizer and double centralizer theorems for simple finite dimensional subalgebras of the associative algebra of all linear transformations on a vector space ([15]) also hold for simple finite dimensional Jordan sub-

This paper was originally accepted by the Transactions of the American Mathematical Society, Received July 10, 1957, by Trans. Amer. Math. Soc. Most of the material of this paper is contained in the author's doctoral dissertation, Yale University, 1956. Some of the results were presented to the Society in October, 1955 and April, 1957. The author is a National Science Foundation Postdoctoral Fellow, 1956-57.