# ON THE THEORY OF $(m, n)$-COMPACT TOPOLOGICAL SPACES

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In a recent paper I introduced the following generalization of the notion of compactness:

*A topological space $X$ is $(m, n)$-compact if from every open covering $\{O_i\}$ $(i \in I)$ of $X$ whose cardinality card $I$ is at most $n$ one can select a subcovering $\{O_{i_j}\}$ $(j \in J)$ of $X$ whose cardinality* card $J$ *is at most $m$.*

A similar definition was introduced earlier by P. Alexandroff and P. Urysohn [1]. If no inaccessible cardinals exist between $m$ and $n$ the two definitions are equivalent. The present definition has the advantage that in applications the question of the existence of inaccessible cardinals does not generally come up. The basic results on $(m, n)$-compact spaces were published by me in [8] and a detailed study of generalized compactness in the Alexandroff-Urysohn sense was made by Yu. M. Smirnov in [14] and [15]. The special case $m = \omega$ and $n = \infty$ was first studied much earlier by C. Kuratowski and W. Sierpinski in [13] and [10]. These spaces are generally known as Lindelöf spaces.

The present paper contains four types of results on $(m, n)$-compactness which were obtained since the publication of [8]. The problems and the principal results are stated in the beginnings of the individual Sections 1, 2, 3, and 4.

The following notations will be used: $\overline{A}$ and $A^i$ denote the closure and the interior of the set $A$. The symbols $O$ and $C$ stand for open and closed sets, respectively. $\phi$ denotes the empty set. $N_x$ is an arbitrary neighborhood of the point $x$ and $O_x$ denotes any open set containing $x$. Filters are denoted by $\mathscr{F}$, nets by $(x_d)$ $(d \in D)$ where $D$ stands for the directed set on which the net is formed. The set of adherence points of $\mathscr{F}$ is denoted by adh $\mathscr{F}$. Similarly the set of adherence points of a net is denoted by $\mathrm{adh}(x_d)$. The set of limit points is denoted by $\lim \mathscr{F}$ and $\lim (x_d)$, respectively. A topological space $X$ is called normal if for any pair of disjoint closed sets $A$ and $B$ there exist disjoint open sets $O_A$ and $O_B$ such that $A \subseteqq O_A$ and $B \subseteqq O_B$.

Uniform structures for a set $X$ will be denoted by $\mathscr{U}$. The symbol $U[x]$ stands for "the vicinity $U \in \mathscr{U}$ evaluated at $x \in X$" so that $U[x] = [y : (x, y) \in U]$. The composition operator is denoted by $\circ$ and so $U \circ V$ consists of those ordered pairs $(x, z) \in X \times X$ for which there is a $y \in X$ with $(x, y) \in U$ and $(y, z) \in V$.

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