EQUIVALENCE AND PERPENDICULARITY OF GAUSSIAN PROCESSES

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1. Introduction. In [6] S. Kakutani showed that if one has equivalent probability measures μ_i and ν_i on the σ-field $\mathscr{S}_i$ of subsets of a set Ω_i, $i = 1, 2, \cdots$, and if μ and ν denote respectively the infinite product measures $\bigotimes_{i=1}^{\infty}\mu_i$ and $\bigotimes_{i=1}^{\infty}\nu_i$ on the infinite product σ-ring generated on the infinite product set Ω, then μ and ν are either equivalent or perpendicular, and he obtained necessary and sufficient conditions for equivalence to occur. The theorem here shown may be regarded as a generalization of a case of the Kakutani theorem.

Similar dichotomies have revealed themselves in the study of Gaussian stochastic processes. C. Cameron and W. T. Martin proved in [2] that if one considers the measures induced on path space by a Wiener process on the unit interval, then if the variances of the processes are different the measures are perpendicular. This sort of result was generalized by U. Grenander, starting from the viewpoint of statistical estimation, and utilizing a Karhunen representation for the processes involved. A wider sufficient condition for perpendicularity of the measures induced on path space by continuous Gaussian processes on the unit interval was obtained by G. Baxter in [1]. Cameron and Martin also examined the effect on the induced measure of taking certain types of affine transformations of a Wiener process (see [3], [4]). I. E. Segal extended their results in [8], and made the situation more transparent by use of his notion of "weak distributions", and in a large class of cases got conditions for equivalence.

In the present note it is shown that the equivalence-or-perpendicularity dichotomy holds in general for pairs of measures induced by Gaussian stochastic processes, and Segal's necesssary and sufficient conditions for equivalence are extended to cover the case of nonzero mean. It has been pointed out to the author by C. Stein that one could also give a proof, in the case of zero mean, by use of the techniques of statistical testing of hypotheses.

2. Several lemmas. All Hilbert spaces mentioned will be over the reals.

Definition 1. An operator T from Hilbert space $\mathbf{H}$ to Hilbert space $\mathbf{K}$ will be called on *equivalence* operator if

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