THE MAXIMAL IDEALS OF CERTAIN FUNCTION ALGEBRAS

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1. Introduction. In this paper we discover the space of maximal ideals for each Banach algebra of the following concrete type. Select an open subset G of S, the compactified complex plane, and let $H(G)$ be the class of complex functions continuous on S and moreover holomorphic on G. This is a Banach algebra, and its space of maximal ideals is shown below to be precisely S, except in that case in which G is so large as to force $H(G)$ to consist only of constant functions.

Algebras of this type were introduced and studied by J. Wermer [4] and W. Rudin [3]. Wermer pointed out that $H(G)$ need not reduce to the constants even if $S - G$ is required to be (merely) an arc. He also showed distinct points of S determined distinct maximal ideals. Rudin raised the question as to the space of maximal ideals.

K. M. Hoffman, reporting (April 18, 1958, Symposium on Banach algebras and Harmonic analysis) on work by I. M. Singer and himself jointly, showed that the space of maximal ideals of $H(G)$ is S when $S - G$ has positive upper density at each of its points. On the following day, H. L. Royden's proof was presented in which the same desired conclusion was obtained if $S - G$ has dimension zero. Our technique may be regarded as a refinement of Hoffman and Singer's.

Our methods apply equally easily to more general, although perhaps less interesting, algebras. Let Z be a compact subset of S, and let G be an open subset of Z. Let $H(G/Z)$ be the functions continuous on Z and holomorphic on G. Then Z is the space of maximal ideals, unless the algebra reduces to the constants.

For some algebras in this larger class, the problem can also be solved by an appeal to Mergelyan's theorem [5], namely for those $H(G/Z)$ where $Z \neq S$ and G is the interior of Z.

2. An approximation theorem. Let Z be a Borel set in the extended complex plane. Let G be an open set included in Z. We denote by $H(G/Z)$ the class of complex-valued functions which are defined, continuous, and bounded on Z, and are holomorphic on G. $H(G/Z)$ is evidently a Banach algebra with unit, providing that for each $f \in H(G/Z)$ the norm is defined by

$$\|f\| = \sup_{\zeta \in Z} |f(\zeta)| .$$

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