ANALYTIC CONTINUATION OF MEROMORPHIC FUNCTIONS IN VALUED FIELDS

J. B. ROBERTS

In this paper* we consider analytic continuation of power series by matrix methods in arbitrary fields complete with respect to a valuation. In the complex field continuation can generally be achieved by a formal expansion of the given power series about a point in its circle of convergence. The new series (with power series coefficients) generally exists and converges over a circle extending beyond the circle of convergence of the original series.

When the field is non-Archimedean however the new circle of convergence is always contained in the old. Hence in this case we need have recourse to a summability method. In this paper we consider a certain class of matrix methods which can be applied to the power series coefficients appearing in the formal expansion of the original series about points outside the original circle of convergence. The methods will be applicable in Archimedean or non-Archimedean fields.

The work here is based upon Chapter 3 of the author's PhD dissertation written under the direction of Prof. G. K. Kalisch at the University of Minnesota in 1955.

1. Notations and definitions. Throughout the paper k shall be a field which is complete with respect to a valuation, denoted by $|\ |$. Unless stated explicitely to the contrary the valuation may be either Archimedean or non-Archimedean. It is useful to note that, by a theorem of Ostrowski, if the valuation is Archimedean then k is topologically isomorphic with the real or complex numbers.

We shall designate the collection of all infinite series with terms in k by S. Further we introduce an operation, the Cauchy product, into S. If $C = \sum_{i=0}^{\infty} c_i$ and $C' = \sum_{i=0}^{\infty} c'_i$ are in S then the Cauchy product CC' is defined by

$$CC' = \sum_{i=0}^{\infty} \sum_{j=0}^{i} c_j c'_{i-j} .$$

This product is clearly in S; so S is closed relative to this multiplication.

The subset of S consisting of all unconditionally convergent series will be denoted by T. When k is non-Archimedean T coincides with the

* This paper was originally accepted by the Trans. Amer. Math. Soc., received by the editors of the Trans. Amer. Math. Soc. January 16, 1957, in revised form April 18, 1958. The author wishes to express his thanks to the referee who through his extensive comments on the first version has changed the character of the whole paper and has increased its generality in certain respects.