A GENERALIZATION OF ATOMIC BOOLEAN ALGEBRAS

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1. Introduction. A Boolean algebra B is called atomic if every non-zero element of B contains an atom. A variant of this definition is the equivalence: B is atomic if and only if B contains a dense (i.e., coinitial in $B - \{0\}$) subset which is totally unordered. In this paper, we will investigate the properties of Boolean algebras which contain dense subsets of somewhat more general order type than the totally unordered sets.

DEFINITION 1.1. Let α be an infinite cardinal number. A partially ordered set P will be called *α-compact* if P is closed under finite meets, contains a zero element and satisfies the condition that if $M \subseteqq P$ has cardinality $\leqq \alpha$ and no finite subset of M has zero meet, then M has a non-zero lower bound in P.

The use of the term "compact" is of course motivated by the topological analogy.

DEFINITION 1.2. A Boolean algebra B will be called *α-atomic* if B contains a dense subset which is α-compact.

Since a totally unordered set becomes α-compact (for all α) if a zero element is adjoined to it, an atomic Boolean algebra is α-atomic for all cardinals α.

The organization of the paper is as follows. Section two is devoted to the construction of examples of α-atomic Boolean algebras. In section three, some properties of α-atomic Boolean algebras are proved. Section four presents a representation theorem for α-atomic algebras.

Throughout the paper, α will denote a fixed infinite cardinal number. The abbreviation α-B.A. will be used for α-complete Boolean algebra. The terms α-subalgebra, α-ideal, α-homorphism, α-field, etc. have their usual meanings. Thus, an α-homorphism of an α-B.A. is a homomorphism preserving α-joins; an α-subalgebra of an α-B.A. is a subalgebra closed under formation of α-joins in the enveloping algebra. It is sometimes convenient to use the symbol ∞ in place of α with the meaning that the corresponding property is to hold for all cardinals.

The lattice operations of join, meet and complement are designated by $\vee$, $\wedge$, and ($'$) respectively. The symbols 0 and u denote the zero and unit in a Boolean algebra. Set operations are indicated by rounded symbols: $\cap$, $\cup$ and $\subseteqq$ stand for intersection, union and inclusion

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