DIRICHLET MULTIPLICATION IN LATTICE POINT PROBLEMS. II

J. P. TULL

1. The author [8]* has given a theorem in which we assume two functions A and B have asymptotic formulae of the form

$$(1)\qquad A(x) = \sum_{\mu=1}^{h} x^{\alpha_\mu} P_\mu(\log x) + O\{x^\alpha \log^l(x+1)\}$$

and upper estimates $O\{x^\rho \log^m(x+1)\}$ on their total variations. We then conclude that their Stieltjes resultant C satisfies a formula similar to (1). The α_μ are complex numbers, the P_μ are polynomial functions, and we give an explicit formula for the error term in the resultant in terms of the given parameters.

In this paper we shall give a generalization of the above-mentioned result which will cover a wider class of lattice point problems.

2. Given two functions A and B defined for $x \geqq 1$, of bounded variation on each bounded interval, we call the Stieltjes resultant of A by B any function C such that

$$(1)\qquad C(x) = \int_1^x A(x/u)dB(u)$$

wherever the integral exists and for all x either.

$$(2)\qquad \lim_{h\to 0+} C(x-h) \leqq C(x) \leqq \lim_{h\to 0+} C(x+h)$$

or

$$(3)\qquad \lim_{h\to 0+} C(x+h) \leqq C(x) \leqq \lim_{h\to 0+} C(x-h)\ .$$

Note that there are at most countably many x for which the integral (1) does not exist, namely those $x = ab$ where a is a discontinuity of A and b is a discontinuity of B. Note further that if $A(1) = B(1) = 0$ then the Stieltjes resultant is a commutative binary operation:

$$(4)\qquad \int_1^x A(x/u)dB(u) = \int_1^x B(x/u)dA(u)\ .$$

Widder [9] gives a slightly more restrictive definition of Stieltjes resultant, however, his requirement that $A, B,$ and C be "normalized" is unnecessary.

Received August 19, 1958. Excerpt from dissertation for the degree Doctor of Philosophy, University of Illinois, 1957.

* Note that in the proof of [8] Theorem 2, no use was made of the assumption α, β, ρ, τ are non-negative. Thus that assumption may be deleted from the theorem.