A MAXIMAL PROBLEM IN HARMONIC ANALYSIS, II

I. I. HIRSCHMAN, JR.

1. Introduction. Let G be a compact topological group with elements x, x_0, etc. We denote by dx the Haar measure of G normalized by the condition that the measure of G is 1. Let the matrices

$$(1) \qquad [g(\alpha, i, j, x)]_{i,j=1}^{r(\alpha)} \qquad \alpha \in A$$

be a complete set[1] of inequivalent unitary representations of G. We recall that this implies that[2]

$$\int_G g(\alpha, i, j, x) g(\beta, k, l, x)^* dx = \frac{\delta(\alpha, i, j; \beta, k, l)}{r(\alpha)} .$$

Here $\delta(\alpha, i, j; \beta, k, l)$ is 1 if $\alpha = \beta, i = k$ and $j = l$; otherwise it is zero. Further if $f(x) \in L^2(G)$ and if

$$c(\alpha, i, j, f) = \int_G f(x) g(\alpha, i, j, x)^* dx ,$$

then

$$(2) \qquad \left\{ \sum_\alpha r(\alpha) \sum_{i,j=1}^{r(\alpha)} |c(\alpha, i, j, f)|^2 \right\}^{1/2} = \|f\|_2 .$$

Let $1 < p \leqq 2, 1/p + 1/q = 1$. The object of the present paper is to demonstrate the inequalities

$$(3') \qquad \left\{ \sum_\alpha r(\alpha)^{2-q/2} \left[\sum_{i,j=1}^{r(\alpha)} |c(\alpha, i, j, f)|^2 \right]^{q/2} \right\}^{1/q} \leqq \|f\|_p ,$$

$$(3'') \qquad \left\{ \sum_\alpha r(\alpha)^{2-p/2} \left[\sum_{i,j=1}^{r(\alpha)} |c(\alpha, i, j, f)|^2 \right]^{p/2} \right\}^{1/p} \geqq \|f\|_q ,$$

and to determine for $p \neq 2$ all cases in which equality occurs. (If $p = q = 2$ then (3′) and (3″) reduce to (2) and equality holds for every f). The inequalities (3′) and (3″) are an extension to compact groups of the Young-Hausdorff-Riesz inequalities for Fourier series. The corresponding problem for locally compact Abelian groups has been discussed by E. Hewitt and the author in [2], and the present paper may be considered as a continuation of [2]. Closely related results are also contained in a paper of A. Calderón and A. Zygmund [1].

Note that the $r(\alpha) \times r(\alpha)$ matrix $[g(\alpha, i, j, x)]$ is not uniquely

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[1] For the definitions of the group theoretic terms used here see [3].

[2] If γ is a complex number then γ^* denotes its conjugate.