A BOUND FOR THE ORDERS OF THE COMPONENTS OF A SYSTEM OF ALGEBRAIC DIFFERENCE EQUATIONS

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1. The object of this paper is to obtain a bound for the orders of the components of a system of algebraic difference equations, *each component of which is of dimension zero*. In the analytic case, this roughly amounts to determining the maximum number of arbitrary functions of period unity which each corresponding manifold can possess.

2. We deal with difference polynomials in n indeterminates $y_1, \cdots, y_n$ having coefficients in an inversive difference field, $\mathscr{F}$, of characteristic zero. Transforms are denoted by means of a second subscript appended to Latin letters having a single subscript. Thus, for example, $A_{3,4}^{(2)}$ denotes the fourth transform of $A_3^{(2)}$. The symbol $\mathscr{F}\{y_1, \cdots, y_n\}$ denotes the ring of difference polynomials in the indeterminates $y_1, \cdots, y_n$. The perfect difference ideal generated by a system Φ of difference polynomials is designated $\{\Phi\}$. Unless there is a possibility for confusion, the term "ideal" is used for the longer "reflexive difference ideal". It is well known that every perfect ideal is the intersection of a finite number of prime ideals, none of which contain any other, [4]. As in ordinary or in differential algebra, these prime ideals are termed *components* of the decomposition of the perfect ideal.

If Λ is a prime ideal in $\mathscr{F}\{u_1, \cdots, u_q; y_1, \cdots, y_p\}$, then the u_i are said to constitute a *parametric set of indeterminates*, or briefly *parameters*, of Λ if

(1) Λ contains no nonzero difference polynomial in the u_i alone;

(2) for each k, $1 \leqq k \leqq p$, there exists in Λ a nonzero difference polynomial in y_k and $u_1, \cdots, u_q$.

It is shown in [1, p. 141] that all parametric sets of a given reflexive prime difference ideal Λ contain the same number of parameters. This number is known as the *dimension* of Λ, and is briefly denoted *dim* Λ. If the prime ideal has no parameters, we say its dimension is zero.

By the *order* of a prime ideal Λ in $\mathscr{F}\{y_1, \cdots, y_n\}$, we mean the algebraic dimension of Λ, that is $\partial^0 \mathscr{F}(\eta_1, \cdots, \eta_n; \eta_{11}, \cdots, \eta_{n1}; \eta_{12}, \cdots, \eta_{n2}; \cdots)/\mathscr{F}$ or $\partial^0 \mathscr{F} < \eta_1, \cdots, \eta_n > / \mathscr{F}$, where $\eta_1, \cdots, \eta_n$ is a generic zero of Λ.

A system of difference (differential) polynomials in $\mathscr{F}\{y_1, \cdots, y_n\}$ is said to be of *type* $(r_1, \cdots, r_n)$ if $r_1, \cdots, r_n$ are the maximum orders of the transforms (derivatives) of $y_1, \cdots, y_n$ respectively that appear in the system.

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