MARKOV OPERATORS AND THEIR ASSOCIATED SEMI-GROUPS

R. K. Getoor

1. Introduction. The present paper is an extension and continuation of our earlier paper " Additive Functionals of a Markov Process " [5] which will be referred to in the sequel as AF. Roughly speaking we consider a temporally homogeneous Markov process, $x(t)$, in a locally compact, separable, metric space and certain other processes derived from it. We always assume $x(t)$ has right continuous paths and we consider processes obtained by stopping $x(t)$ at the boundary of an open set, G, and subjecting $x(t)$ to a local " death rate ", $V(x)$, in G. Our main study is the relationships between the infinitesimal generators of certain semi-groups naturally associated with these processes.

Actually we use a function space approach to stochastic processes and so our results are of an analytic nature (i. e. relations between the transition probabilities and infinitesimal generators) rather than of a measure theoretic nature (i. e. statements above sample functions, etc.). The use of a function space approach simplifies many measure theoretic difficulties associated with conditional probabilities and expectations, but introduces the difficulty that if G is open then $G(t) = \{x(\cdot): x(\tau) \in G; 0 \leqq \tau \leqq t\}$ is not in general measurable with respect to the σ-algebra $\mathfrak{B}(\mathfrak{X})$ defined in § 2. It is known [7] that under certain restrictions (implied by our assumptions in § 2) $G(t)$ is measurable with respect to the appropriate completion of $\mathfrak{B}(\mathfrak{X})$. However, we do not choose to complete $\mathfrak{B}(\mathfrak{X})$ as this introduces the other difficulties mentioned above; instead we consider the set $\{x(\cdot): x(\tau) \in \bar{G}; 0 \leqq \tau \leqq t\}$ ($\bar{G}$ denotes the closure of G) which is obviously in $\mathfrak{B}(\mathfrak{X})$ and impose a regularity condition on G that insures us that these two sets are roughly the same. (Theorem 2.1 and the ensuing development.)

In § 2 we develop the preliminary machinery that is needed throughout the remainder of the paper. We show in § 2 that all the results of AF are valid without the assumption (P_3) of AF. In § 3 we investigate the behavior at the boundary of G of the semi-groups introduced in § 2. In § 4 we consider the special case in which the infinitesimal generator of the semi-group associated with $x(t)$ is a local operator. The results of this section also extend and complement those of AF. In the remaining three sections of the paper we study the spectral properties of the semi-groups introduced in the earlier part of the paper.

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