ALMOST LOCALLY PURE ABELIAN GROUPS

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0. **Introduction.** It is the purpose of this paper to introduce and to give a preliminary investigation of almost locally pure Abelian groups [see definition 1]. For primary groups the concept of almost locally pure Abelian group coincides with that of no elements of infinite height [Theorem 9].

1. DEFINITION. A group (= Abelian group), G, is *almost locally pure* (hereafter abbreviated a.1.p.) if for every finite set of elements $g_1, \cdots, g_n$ of G there exists a finitely generated pure subgroup, P, of G which contains $g_1, \cdots, g_n$.

2. EXAMPLES. Direct sums of cyclic groups are clearly a.1.p. The complete direct sum of copies of the integers is a.1.p. since by [1] every finite subset is contained in a completely decomposable direct summand and each such summand is free of finite rank.

3. REMARK. If one defines a group G to be locally pure if every finite subset generates a pure subgroup, then it is easy to see that G is a direct sum of cyclic groups of prime order, for various primes.

4. THEOREM. *A direct sum of* a. 1. p. *groups is* a.1.p.

Proof. Let $G = \sum_\alpha \oplus H_\alpha$, where $\oplus$ denotes the weak direct sum, and where H_α is a.1.p. for all α. Let $g_1, \cdots, g_n$ be in G. Now let H_β be a summand in which some g_i has a non-zero component, and consider the components $g_{\beta_1}, \cdots, g_{\beta n}$ of $g_1, \cdots, g_n$ in H_β. In each such H_β (there are only a finite number) there exists a finitely generated pure subgroup P_β containing $g_{\beta_1}, \cdots, g_{\beta_n}$. Then $\sum_\beta \oplus P_\beta$ is a finitely generated pure subgroup containing $g_1, \cdots, g_n$.

5. THEOREM. *If G is* a.1.p., *if K is a subgroup of G, and if for every finite set of elements $g_1, \cdots, g_n$ of G, there exists a pure subgroup, P, of G such that the group generated by K and $g_1, \cdots, g_n$ is a subgroup of P and P/K is finitely generated, then G/K is* a.1.p.

If G and G/K are a.1.p., *where K is pure in G, then for every finite set of elements $g_1, \cdots, g_n$ of G, there exists a pure subgroup, P, of G such that the group generated by K and $g_1, \cdots, g_n$ is a subgroup of P, and P/K is finitely generated.*

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