THE SUSPENSION OF THE GENERALIZED PONTRJAGIN COHOMOLOGY OPERATIONS

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1. The main theorem. In a previous paper [9] I have defined a sequence of new cohomology operations, called the *generalized Pontrjagin operations*. These operations use as coefficient groups the summands of a certain type of graded ring: namely, *a ring with divided powers* (defined by H. Cartan in [1]), which is termed a Γ-ring in [9]. Let $A = \sum_k A_k$ be a ring with divided powers such that each summand A_k is a cyclic group of infinite or prime power order; we termed such rings *p-cyclic* in [9]. Then, the Pontrjagin operations are functions

$$\mathfrak{P}_t\colon H^{2n}(X;\ A_{2k}) \longrightarrow H^{2tn}(X;\ A_{2tk}) \quad (k,\ n > 0;\ t = 0,\ 1, \cdots)$$

where $H^q(Y,\ B;\ G)$ denotes the qth (singular) cohomology group of the pair $(Y,\ B)$ with coefficients in the group G.

Let C be a cohomology operation relative to integers r, s and coefficient groups G, Π. That is, C is a natural transformation

$$C\colon H^r(Y,\ B;\ G) \longrightarrow H^s(Y,\ B;\ \Pi)\ .$$

With each operation C we associate a second operation, $S(C)$, called the *suspension* of C. $S(C)$ is a natural transformation

$$H^{r-1}(Y,\ B;\ G) \longrightarrow H^{s-1}(Y,\ B;\ \Pi)\ ;$$

its definition is given in § 3.

The purpose of this note is to determine $S(\mathfrak{P}_t)$, where $\mathfrak{P}_t$ is the generalized Pontrjagin operation. In order to state our result concerning $S(\mathfrak{P}_t)$, we need an additional cohomology operation, the Postnikov square (see [3], [10]). This was defined in [9], but only for a restricted class of coefficient groups. In this paper we will define the Postnikov square as a cohomology operation

$$\mathfrak{p}\colon H^q(Y,\ B;\ A_{2k}) \longrightarrow H^{2q+1}(Y,\ B;\ A_{4k})\ , \qquad (q,\ k > 0)$$

where A_{2k} is an even summand of a p-cyclic ring with divided powers.

We now may state the main result of the paper.

THEOREM I. *For any cohomology operation C, let $S(C)$ denote the suspension of the operation C. Then,*

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