SILOV TYPE C ALGEBRAS OVER A CONNECTED LOCALLY COMPACT ABELIAN GROUP

ALFRED B. WILLCOX

A certain class of commutative Banach algebras of functions on a compact abelian group has been studied by G. E. Silov [6]. His algebras, which he calls homogeneous rings, are partially characterized by the property of containing arbitrary translates of elements. The most interesting examples are various algebras of complex functions on the circle or torus of any dimension with various differentiability properties and algebras of continuous functions on a compact abelian group which have absolutely convergent Fourier series. Silov's results have been extended by Mirkil [5] to algebras over non-abelian compact groups. We present here some results which generalize parts of the theory to translation closed algebras over connected locally compact abelian groups. The major problem in an extension in this direction centers about a replacement for the type of classical Fourier analysis for continuous functions on compact groups which has no satisfactory analog even in the abelian non-compact case. Our approach to this problem is to recapture *locally* some of the compact case when it becomes necessary. This approach makes it necessary to add to Silov's conditions various additional assumptions. Nevertheless, a considerable portion of the theory survives; enough, in fact, to include analogs of all the interesting examples from the compact case. In § 1 we present the basic construction on which the structure theorems of § 2 are based. In § 3 various examples are discussed. It will be assumed that the reader is familiar with the general theory of commutative regular Banach algebras. An account assuming an identity can be found in [6]. The results extend easily to algebras without identity. Such extensions can be found in [2], [3], [4], or, for certain non-commutative algebras, in [8].

1. In this section we describe a method of constructing a Banach algebra from the following ingredients:

(i) a connected locally compact abelian group G,

(ii) a primary commutative Banach algebra K with identity, maximal ideal Q, and norm $|\cdot|$, and

(iii) a homomorphism ω of the character group $\hat{G}$ of G into the coset of the identity in K modulo Q.

By well-known structure theorems [7, section 29] $G = E_p \times G_c$ where E_p is the p-dimensional vector group and G_c is compact abelian. From

Received August 8, 1958, and in revised form February 27, 1959.