HIGHER DIMENSIONAL CYCLIC ELEMENTS

JOHN GARY

Introduction. Whyburn, in 1934, introduced the higher dimensional cyclic elements [5]. He gave an analysis of the structure of the homology groups of a space in terms of its cyclic elements. His results were for finite dimensional spaces, and he used the integers modulo two as the coefficient group. Puckett generalized some of Whyburn's results to compact metric spaces [3]. Simon has shown that if E is a closed subset of a compact space M, which contains all the $(r-1)$-dimensional cyclic elements of M, then $H^r(E) \approx H^r(M)$[4]. He also obtained a direct sum decomposition of $H^r(M)$ using the cyclic elements of M. We will extend some of these results.

The properties of zero-dimensional cyclic elements in locally connected spaces, and the relation of these cyclic elements to monotone mappings, is basic in the applications of zero-dimensional cyclic element theory. We shall give some counter-examples concerning the generalization of these properties to higher dimensional cyclic elements.

1. Preliminaries. Throughout this paper M will always denote a compact Hausdorff space. We shall use the augmented Cech homology and cohomology with a field as coefficient group. Results stated in terms of cohomology may be given a dual expression in terms of homology by means of the dot product duality for the Cech theory.

DEFINITION 1.1. A T_r set in M is a closed subset T of M such that $H^r(K) = 0$, for all closed subsets K of T.

DEFINITION 1.2. An E_r set in M is a non-degenerate subset of M which is maximal with respect to the property that it can not be disconnected by a T_r set of M.

The proofs of Lemmas 1.3 through 1.9 can be found in the papers by Whyburn [5] and Simon [4]. The proofs given by Whyburn are for subsets of Euclidean space, but they can be carried over to our case without difficulty.

LEMMA 1.3. *Let K be a subset of M which can not be disconnected by a T_r set. If $M = M_1 \cup M_2$, T_r-separated (by this we mean M_1 and M_2 are proper closed subsets and $M_1 \cap M_2$ is a T_r set), then $K \subset M_1$ (or, $K \subset M_2$).*

Received December 24, 1958. The results of this paper are contained in the authors doctoral dissertation, University of Michigan, 1956. The author wishes to thank Professor R. L. Wilder for his advice and encouragement.