PROBLEMS IN SPECTRAL OPERATORS

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Introduction. An important problem in the theory of spectral operators in Banach spaces initiated by N. Dunford [5; 6] is that of deciding whether the linear operators of the types encountered in analysis are spectral. Various conditions for spectral operators have been given in [5], but further research is needed in order to apply them to specific cases. J. Schwarz [11] has shown that a class of operators arising from, not necessarily self adjoint, integro-differential boundary-value problems consists of spectral operators. The present investigation originated in a problem on stationary sequences in Banach spaces which led to the study of unitary operators, namely linear isometries of the space onto itself, from this point of view. Accordingly, attention was focused on the class of unitary operators, and the limitations imposed on the operators under study were designed to include it.

Section 1 contains a summary of definitions and results from [5; 6]. A distinction, significant only in non-reflexive spaces, is made between spectral and merely prespectral operators according to the topology in which σ-additivity of the resolutions of the identity is required. As shown in § 2, a resolution of the identity of a prespectral operator uniquely determines the resolutions of the identity of its spectral restrictions. A simple example shows how this can be used to prove that certain operators are not spectral.

Known results are combined in § 3 to yield a necessary condition for spectral operators of scalar type, which involves only the norms of rational functions of the operators. If the space is reflexive and the spectrum an R-set [1, p. 397], the condition is also sufficient. Using the results of § 2 this condition is localised to "cyclic" subspaces generated by single elements. A much more general approach to localization, via the notion of vector measures associated with the operator, is expounded in [3]. It is felt though that the present considerations retain their interest owing to the explicit conditions given. The method of [3] also implies the results of § 2 on restrictions for the case of a reflexive space. Section 3 ends with some characterizations of finite dimensional cyclic subspaces.

The above results are specialized in § 4 to unitary operators which, if the space is reflexive, satisfy all the subsidiary conditions. As a corollary it follows that in a reflexive space a unitary operator is spectral

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