A CLASS OF HYPER-*FC*-GROUPS

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1. Introduction. An element g of an arbitrary group G is called an FC element if it has a finite number of conjugates in G. The set of all FC elements of G forms a characteristic subgroup H of G (see Baer [1]). The *upper FC-series* of G, introduced by Haimo [4] as the *FC-chain,* may be defined by

$$H_0 = \{1\} ,$$
$$H_{i+1}/H_i = H(G/H_i) ,$$

the subgroup of all FC elements of G/H_i. The upper FC-series is continued transfinitely in the usual way, by defining

$$H_\alpha = \bigcup_{\beta<\alpha} H_\beta ,$$

when α is a limit ordinal. If $H_\gamma = G$, but $H_\delta \neq G$, for all $\delta < \gamma$, we say that the group G is *hyper-FC of FC-class* γ, following McLain [7].

A group G in which the transfinite upper central series

$$\{1\} = Z_0 \leq Z_1 \leq \cdots \leq Z_\alpha \leq \cdots$$

reaches the whole group is called a ZA-group (Kurosh [6]), and we may say that G has class α if $Z_\alpha = G$, but $Z_\beta \neq G$, for all $\beta < \alpha$. Glushkov [3] and McLain [7] have given constructions for a ZA-group of any given class. The main object of this note is to construct groups of given FC-class.

2. Constructions and proofs.

DEFINITION. We say that a group G is of type Q_α if

1. G has FC-class α, with upper FC-series
$\{1\} = H_0 \leq H_1 \leq \cdots \leq H_\alpha = G$,

2. $H_{\gamma+1}/H_\gamma$ is infinite, for all $\gamma < \alpha$, and

3. $H_{\gamma+1}/H_\gamma$ has the unit subgroup for its centre, for all $\gamma < \alpha$.

Thus the group with only one element is of type Q_0, and, in constructing a group G of type Q_α, we may assume the existence of a group G_β of type Q_β, for each $\beta < \alpha$. If α is a limit ordinal, we define G to be the ordinary (restricted) direct product of the groups G_β, for all $\beta < \alpha$. Then G has the properties $1 - 3$, and thus has type Q_α. For the case $\alpha = \beta + 1$ we shall construct G by 'wreathing' the regular

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