CHARACTERIZATIONS OF TREE-LIKE CONTINUA

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1. Introduction. It has been conjectured by J. R. Isbell that every one dimensional continuum with trivial Čech homology (arbitrary coefficient group) is tree-like. In this note we give an example showing the conjecture is false. Moreover, the example has the Čech homology groups, the Čech cohomology groups, and the Čech fundamental group (see [3]) of a point. Also, the example cannot be mapped essentially onto a circle, but can be mapped essentially onto a "figure 8". We precede the example with two characterizations of tree-like continua.

2. Preliminaries. Throughout this note by a continuum we will mean a compact connected metric space and unless otherwise specified by a complex we will mean a finite complex. Also, by a linear graph we will mean a one dimensional connected complex.

For this section let X be any one dimensional continuum, K be any linear graph, and $\mathscr{C}(X)$ be the collection of all essential finite open covers of order two of X. For $U \in \mathscr{U} \in \mathscr{C}(X)$ let $\mathscr{N}(\mathscr{U})$ denote the nerve (see page 68 of [5]) of $\mathscr{U}$ and $\sigma(U)$ denote that vertex in $\mathscr{N}(\mathscr{U})$ corresponding to U. Note that for any $\mathscr{U} \in \mathscr{C}(X)$, $\mathscr{N}(\mathscr{U})$ is a linear graph. Where $\mathscr{U} \in \mathscr{C}(X)$ and $x \in X$ let $\Delta(\mathscr{U}, x)$ be the simplex in $\mathscr{N}(\mathscr{U})$ which has as vertices the collection of all $\sigma(U)$ such that $x \in U \in \mathscr{U}$. Where $\mathscr{U} \in \mathscr{C}(X)$, a continuous function f from X to $\mathscr{N}(\mathscr{U})$ is said to be a $\mathscr{U}$-canonical mapping provided that $f(x) \in \Delta(\mathscr{U}, x)$ for all $x \in X$. Where f is a continuous function from X to K, let $\mathscr{L}(f)$ be the collection of all non-empty inverse images under f of open stars of vertices in K. Note that $\mathscr{L}(f) \in \mathscr{C}(X)$. Where f is a continuous function from X to K let f' be that simplicial mapping from $\mathscr{N}(\mathscr{L}(f))$ to K which satisfies the condition.

$$f'(\sigma(f^{-1}[\text{open star of } v])) = v$$

for all vertices v in K such that

$$f^{-1}[\text{open star of } v]$$

is non-empty. Where $\mathscr{V}$ and $\mathscr{U}$ are two elements of $\mathscr{C}(X)$ such that $\mathscr{V}$ refines $\mathscr{U}$, a simplicial mapping p from $\mathscr{N}(\mathscr{V})$ to $\mathscr{N}(\mathscr{U})$ is said to be a projection if

$$p(\sigma(V)) = \sigma(U)$$

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