PROJECTIONS ONTO THE SUBSPACE OF COMPACT OPERATORS

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Introduction. The purpose of this paper is to establish the following theorem.

THEOREM. *Suppose U and V are Banach spaces and that there are bounded projections P_1 from U onto X and P_2 from V onto Y. Then there are no bounded projections from the space of bounded operators on U into V onto the closed subspace of compact operators, in the following cases:*

1. *X is isomorphic* [1] *to $\ell^p, 1 \leq p < \infty$; Y is isomorphic to ℓ^q, $1 \leq p \leq q \leq \infty$ or c_0 or c.*
2. *X is isomorphic to c_0; Y is isomorphic to ℓ^∞, c_0 or c.*
3. *X is isomorphic to c; Y is isomorphic to ℓ^∞.*

NOTATION. If X and Y are Banach spaces, $[X, Y]$ is the set of bounded linear operators from X into Y. ℓ^∞ is the set of bounded sequences with the sup norm.

A space X is said to have a countable basis if there is a countable subset of elements of X, called a basis, such that each $x \in X$ is uniquely expressible as

$$x = \sum_{i=1}^{\infty} \xi_i \varphi_i$$

in the sense that

$$\lim_{n \to \infty} \| x - \sum_{i=1}^{n} \xi_i \varphi_i \| = 0 \, .$$

If X and Y are spaces with countable bases (φ_i) and (ψ_i) respectively and A is a bounded linear transformation from X into Y, then A can be represented by an infinite matrix (a_{ij}), with

$$A\varphi_j = \sum_{i=j}^{\infty} a_{ij} \psi_i$$

[2]. In what follows, the basis used for ℓ^p will be given by $\varphi_j = (0, 0, \cdots, 0, 1, 0, 0, \cdots)$ where there is a 1 in the jth place and 0 elsewhere. Similarly for ψ_i. The matrix representations of operators will all be with respect to these bases.

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