NOTE ON ALDER'S POLYNOMIALS

L. CARLITZ

1. Alder's polynomial $G_{M,t}(x)$ may be defined by means of

$$(1)\qquad 1+\sum_{s=1}^{\infty}(-1)^s k^{Ms} x^{\frac{1}{2}s\{(2M+1)s-1\}}(1-kx^{2s})\frac{(kx)_{s-1}}{(x)_s} = \prod_{n=1}^{\infty}(1-kx^n)\sum_{t=0}^{\infty}\frac{k^t G_{M,t}(x)}{(x)_t},$$

where M is a fixed integer ≥ 2 and

$$(a)_t=(1-a)(1-ax)\cdots(1-ax^{t-1}),\ (a)_0=1 .$$

Alder [1] obtained the identities

$$(2)\qquad \prod_{n=1}^{\infty}\frac{(1-x^{(2M+1)n-M})(1-x^{(2M+1)n-M-1})(1-x^{(2M+1)n})}{1-x^n}=\sum_{t=1}^{\infty}\frac{G_{M,t}(x)}{(x)_t},$$

$$(3)\qquad \prod_{n=1}^{\infty}\frac{(1-x^{(2M+1)n-1})(1-x^{(2M+1)n-2M})(1-x^{(2M+1)n})}{1-x^n}=\sum_{t=0}^{\infty}\frac{x^t G_{M,t}(x)}{(x)_t}$$

thus generalizing the well-known Rogers-Ramanujan identities. Singh [2, 3] has further generalized (2), (3); he showed that

$$\prod_{n=1}^{\infty}\frac{(1-x^{(2M+1)n-s})(1-x^{(2M+1)n-2M-1+s})(1-x^{(2M+1)n})}{1-x^n}=\sum_{t=0}^{\infty}\frac{A_s(x,\ t)G_{m,t}(x)}{(x)_t},$$

where the $A_s(x,\ t)$ are polynomials in x.

In a recent paper [4] Singh has proved that

$$(4)\qquad G_{M,t}(x)=x^t \qquad (t\leq M-1).$$

In the present note we give another proof of (4) and indeed obtain the explicit formula

$$(5)\qquad G_{M,t}(x)=\sum_{\substack{Ms\leq t\\ s\geq 0}}(-1)^s\frac{(x)_t}{(x)_s(x)_{t-Ms}}x^{\frac{1}{2}s(s-1)+st}(1-x^s+x^{t-Ms+s})$$

valid for all t.

2. Since

$$(1-kx^{2s})(kx)_{s-1}=(kx)_s+kx^s(1-x^s)(kx)_{s-1},$$

the left member of (1) is equal to

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