POWER CHARACTER MATRICES

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Introduction. In 1956 [4] we gave two classes of matrices whose elements are simple functions of their row and column numbers and whose characteristic roots, inverse, determinant as well as the general element of any power of the matrix can be given explicitly. The elements of these matrices are simple real functions of the real non-principal character χ modulo an odd prime. Such matrices are useful as test matrices in checking out automatic machine programs for general matrices with real elements. In this paper we present corresponding matrices with complex elements which may be used likewise as test matrices. The elements are based on kth power characters χ which are complex roots of unity if $k > 2$.

The general method for finding characteristic roots is the same in both papers and depends on the simple fact that the roots of a polynomial are determined by the sums of like powers of its roots.

All matrices in this paper are square and of order $p-1$ where p is an odd prime.

NOTATION AND DEFINITIONS. Let k be an integer greater than 1. Let $p = kt + 1$ be a prime and let g be a fixed primitive root of p. Let $\alpha = \exp\{2\pi i/k\}$. The kth power character χ, depending on g, is defined by

$$\chi(h) = \begin{cases} 0 \text{ if } p \text{ divides } h \\ \alpha^{\operatorname{ind} h} \text{ otherwise} \end{cases}$$

where $\operatorname{ind} h = \operatorname{ind}_g h$ is the index of h to the base g defined modulo $p-1$ by

$$g^{\operatorname{ind} h} \equiv h (\operatorname{mod} p) .$$

The following well-known properties of χ are simple consequences of our definition of χ and are used many times in the sequel.

$$(1) \qquad \begin{aligned} \chi(h+p) &= \chi(h) \\ \chi(h_1 h_2) &= \chi(h_1)\chi(h_2) \\ \bar{\chi}(h) &= 1/\chi(h) = \chi(\bar{h}) \qquad (h\bar{h} \equiv 1 \pmod p) \\ \chi(h)^k &= \begin{cases} 0 \text{ if } p \mid h \\ 1 \text{ otherwise} \end{cases} \end{aligned}$$

$$(2) \qquad \sum_{h=1}^{p-1} [\chi(h)]^r = \begin{cases} p-1 \text{ if } k \mid r \\ 0 \quad \text{otherwise} . \end{cases}$$

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