INFINITELY REPEATABLE GAMES

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1. Introduction. Blackwell [1] has introduced the concept of approachability in obtaining an analog of the von Neumann minimax theorem for games with vector payoffs. This paper continues the study of this concept. Games with vector payoffs are again two person decision problems with each player having r and s pure strategies respectively but the element of the payoff matrix corresponding to the (i, j) strategy pair is a point $g(i, j)$ in Euclidean N-space. Let C_G denote the convex hull of the rs points $g(i, j)$. Then the problem studied in approachability theory can be stated briefly as follows. If a game with vector payoffs is repeated in time can player I force the average payoff to approach a preassigned closed subset S of C_G with probability approaching 1 as the number of plays becomes infinite?

Because a sequence of games is being considered the rules of play must specify to what extent a player's decision at any stage may depend on past plays. This leads to the natural question of how the class of approachable sets depends on the type of information available to player I. It is specifically this question that is considered in this paper. The problem is formulated mere precisely below.

Let

$$G = \|g(i, j)\|, \qquad 1 \leqq i \leqq r,\ 1 \leqq j \leqq s$$

be an $r \times s$ matrix each element of which is a point in Euclidean N-space and let

$$\mathscr{F} = \|e_{(i,j),k}\|\ , \quad 1 \leqq i \leqq r\ , \quad 1 \leqq j \leqq s\ , \quad 1 \leqq k \leqq t$$

denote an $rs \times t$ matrix such that $0 \leqq e_{(i,j),k}$ (for all $1 \leqq i \leqq r$, $1 \leqq j \leqq s$, $1 \leqq k \leqq t$) and $\sum_{k=1}^{t} e_{(i,j),k} = 1$ (for all $1 \leqq i \leqq r$, $1 \leqq j \leqq s$). A pair $(G, \mathscr{F})$ will determine a game as follows. By a strategy for player I is meant a sequence $f = \{f_n : n = 0, 1, 2, \cdots\}$ of functions where f_n, for $n = 1, 2, \cdots$, is a mapping from the set of n – tuples $(a_1, a_2, \cdots, a_n)$, $a_i \in \{1, 2, \cdots, t\}$, to the set $P = \{(p_1, \cdots, p_r) | 1 \leqq p_i, \sum_1^r p_i = 1\}$, and f_0 is a point in P. A strategy for player II is a sequence of vectors $h = \{h_n : n = 0, 1, 2, \cdots\}$ where $h_n \in Q = \{(q_1, \cdots, q_s) | 0 \leqq q_j, \sum_1^s$

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