A MULTIMOVE INFINITE GAME WITH LINEAR PAYOFF

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1. Introduction. Games can be classified in terms of the number of moves by each player—unimove or multimove—and in terms of the number of choices—finite or infinite—available at each move. The original work of von Neumann [2] on the existence and structure of solutions of games was, in effect, restricted to unimove finite games. Later, Ville [3] proved the existence of optimal strategies for unimove infinite games with continuous payoff function.

Except for games with perfect information, multimove finite games have been analyzed only very recently; and multimove infinite games with an arbitrary number of moves have hardly been touched upon.

In this paper, we analyze a multimove infinite game with a linear payoff function. The game is symmetric in every respect except that the initial conditions of the two players are different. We prove that one player has an optimal pure strategy and that the other player must randomize on the strategies. The optimal strategies and game value are derived.

Although this game had its origin in a military problem concerning allocation of resources among several tasks, it is presented here solely for its mathematical interest. A complete discussion of the military problem and its solution is given in [1].

2. Description of game. We shall analyze the following multimove zero-sum two-person game. At the nth move, or stage of the game, Blue has resources given by the state variable p_n and assigns a value to each of two tactical variables under his control, x_n and u_n, subject to the constraints

$$x_n \geq 0, \quad u_n \geq 0, \quad x_n + u_n \leq p_n \,. \tag{2.1}$$

At the same time, Red has resources given by the state variable q_n and controls the values of the tactical variables y_n and w_n, subject to the constraints

$$y_n \geq 0, \quad w_n \geq 0, \quad y_n + w_n \leq q_n \,. \tag{2.2}$$

Let us number the moves from the end of the game; i.e., the nth move means n moves to the end of the game. The state variables at the $(n-1)$-st move are defined by

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