# AN ANALYTIC PROBLEM WHOSE SOLUTION FOLLOWS FROM A SIMPLE ALGEBRAIC IDENTITY

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**1. Introduction.** It is convenient to describe the point of view of this paper in terms of a very simple example. The unique solution of

$$\frac{dy}{dx} = \lambda\varphi(x)y\ , \qquad y(0) = 1\ , \tag{1.1}$$

where $\varphi(x)$ is a continuous function and $\lambda$ is a parameter, is given by

$$y = \exp\left\{\lambda\int_0^x \varphi(\xi)\,d\xi\right\}\ . \tag{1.2}$$

For any continuous function $\varphi(x)$ define

$$\varphi^+ = \varphi^+(x) = \int_0^x \varphi(\xi)d\xi\ . \tag{1.3}$$

After integrating both sides of the equation in (1.1) and using the notation of (1.3), we find that

$$y = 1 + \lambda(\varphi y)^+ \tag{1.4}$$

has the solution

$$y = \exp(\lambda\varphi^+) = 1 + \lambda\varphi^+ + \lambda^2\varphi^{+2}/2! + \lambda^3\varphi^{+3}/3! + \cdots \tag{1.5}$$

By the method of successive substitutions it is also possible to give a unique solution to (1.4) in the form

$$y = 1 + \lambda\varphi^+ + \lambda^2(\varphi\varphi^+)^+ + \lambda^3(\varphi(\varphi\varphi^+)^+)^+ + \cdots \tag{1.6}$$

Equating coefficients in (1.5) and (1.6) we arrive at the well-known *identities in* $\varphi$

$$\begin{aligned} (\varphi\varphi^+)^+ &= \varphi^{+2}/2! \\ (\varphi(\varphi\varphi^+)^+)^+ &= \varphi^{+3}/3! \\ &\cdots\cdots\cdots\cdots \end{aligned} \tag{1.7}$$

We now wish to focus on the following fact: *All of the identities in* (1.7) *are a consequence of the first identity and the linear property*

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