REGULAR COVERING SURFACES OF RIEMANN SURFACES

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Introduction. The homotopy and homology groups of a given arcwise connected surface are topological invariants. A smooth covering surface F^* is a locally-topological equivalent of its base surface F. Consequently, it is natural that the fundamental and homology groups of F^*, $T(F^*)$ and $H(F^*)$ respectively, should be related to those of F, $T(F)$ and $H(F)$ respectively. In this paper the term homology is always used for the 1-dimensional case. The cover transformations of a covering surface F^* are topological self-mappings such that corresponding points have the same projection on F. These cover transformations form a group which we will denote by $\Gamma(F^*)$. The homology properties of F should influence $\Gamma(F^*)$ by means of the composition of the self-topological mapping and the locally-topological mapping $F^* \to F$.

Section 1 considers the general class of smooth covering surfaces on which there exists a continuation along every arc of the base surface. We refer to such a covering surface as a regular covering surface F^*. A number of results are collected and put into the form in which they are needed to derive the main theorems. The class $\{F^*\}$ is shown to form a complete lattice. Next there is shown a one to one correspondence between all subgroups $N_i \subset T(F)$, such that N_i contains the commutator subgroup N_c of $T(F)$, and the set of all subgroups $H_i \subset H(F)$. This correspondence leads to isomorphisms which relate the associated subgroups.

Section 2 considers a special class of regular covering surfaces $\{F_h^*\}$ in which F_h^* is characterized by the properties that it corresponds to a normal subgroup of $T(F)$ and $\Gamma(F_h^*)$ is Abelian. In our notation these covering surfaces form the class of homology covering surfaces (cf. Kerékjárto [5]). An equivalent characterization of the property that F^* corresponds to a normal subgroup is the assumption that above any closed curve on F there never lie two curves on F^* one of which is closed and the other open. There are derived here for $\{F_h^*\}$ an isomorphism and correspondence theorem which relates subgroups $\Gamma_i \subset \Gamma(F_h^*)$ to quotient groups of $H(F)$ and $T(F)$. The class $\{F_h^*\}$ is shown to form a complete and modular lattice. If the base surface F is an orientable or non-orientable closed surface, with covering surface F_h^*, the rank of $\Gamma(F_h^*)$ is determined in terms of the genus of F and the

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