AN APPROXIMATION THEOREM FOR THE POISSON BINOMIAL DISTRIBUTION

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1. Introduction. Let $x_j; j = 1, 2, \cdots$ be independent random variables such that $\text{Prob}(X_j = 1) = 1 - \text{Prob}(X_j = 0) = p_j$. Let $Q = \mathscr{L}(\Sigma X_j)$ be the distribution of their sum. This kind of distribution is often referred to as a Poisson binomial distribution. For any finite measure μ on the real line let $\|\mu\|$ be the norm defined by

$$\|\mu\| = \sup_f \left\{ \left| \int f d\mu \right| \right\} .$$

the supremum being taken over all measurable functions f such that $|f| \leqq 1$. Let $\lambda = \Sigma p_j$, let $\Sigma p_j^2 = \lambda \varpi$ and let $\alpha = \sup_j p_j$. Finally let P be the Poisson distribution whose expectation is equal to λ.

The purpose of the present paper is to show that there exist absolute constants D_1 and D_2 such that $\|Q - P\| \leqq D_1 \alpha$ for all values of the p_j's and $\|Q - P\| \leqq D_2 \varpi$ if $4\alpha \leqq 1$.

The constant D_1 is not larger than 9 and the constant D_2 is not larger than 16.

Such a result can be considered a generalization of a theorem of Yu. V. Prohorov [9] according to which such constants exist when all the probabilities p_j are equal.

The norm $\|Q - P\|$ is always larger than the maximum distance $\rho(P, Q)$ between the cumulative distributions. For this distance ρ a very general theorem of A. N. Kolmogorov [6] implies that $\rho(P, Q)$ is at most of order $\alpha^{1/5}$. The improvement obtained here is made possible by the smaller scope of our assumptions.

The method of proof used in the present paper is not quite elementary, since it uses both operator theoretic methods and characteristic functions. The relevant concepts are described in § 2.

A completely elementary approach, described in [4] leads to bounds of the order of $3\alpha^{1/3}$ for the distance ρ. Unfortunately, the elementary method does not seem to be able to provide the more precise result of the present paper.

The developments given here were prompted by discussions with J. H. Hodges, Jr. in connection with the writing of [4].

2. Measures as operators. Let $\{\mathfrak{H}, \mathfrak{A}\}$ be a measurable Abelian group, that is, an Abelian group on which a σ-field $\mathfrak{A}$ has been selected

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