SUMMABILITY OF DERIVED CONJUGATE SERIES

B. J. BOYER

1. Introduction. In a recent paper ([3] it was shown that the summability of the successively derived Fourier series of a *CP* integrable function could be characterized by that of the Fourier series of another *CP* integrable function. It is the purpose of the present paper to give analogous theorems for the successively derived conjugate series of a Fourier series.

2. Definitions. The terminology used in [3] will be continued in this paper. In addition let us define:

$$\psi(t) = \psi(t, r, x) = \frac{1}{2}[f(x+t) + (-1)^{r-1}f(x-t)] \tag{1}$$

$$Q(t) = \sum_{i=0}^{[\frac{r-1}{2}]} \frac{\bar{a}_{r-1-2i}}{(r-1-2i)!} t^{r-1-2i} \tag{2}$$

$$g(t) = r!t^{-r}[\psi(t) - Q(t)] \tag{3}$$

The rth derived conjugate series of the Fourier series of $f(t)$ at $t = x$ will be denoted by $D_rCFSf(x)$, and the nth mean of order (α, β) of $D_rCFSf(x)$ by $\bar{S}^r_{\alpha,\beta}(f, x, n)$.

3. Lemmas.

LEMMA 1. *For* $\alpha = 0, \beta > 1$ *or* $\alpha > 0, \beta \geqq 0$, *and* $r \geqq 0$,

$$\bar{\lambda}^{(r)}_{1+\alpha,\beta}(x) = -\pi^{-1}r!(-x)^{r+1} + 0(|x|^{-1-\alpha}\log^{-\beta}|x|) + 0(|x|^{-r-2}) \text{ as } |x| \to \infty .$$

This is a result due to Bosanquet and Linfoot [2].

LEMMA 2. *For* $\alpha > 0, \beta \geqq 0$ *or* $\alpha = 0, \beta > 0$ *and*

$$r \geqq 0,\ x^r\bar{\lambda}^{(r)}_{1+\alpha+r,\beta}(x) = \sum_{i,j=0}^{r} B^r_{ij}(\alpha, \beta)\bar{\lambda}_{1+\alpha+r-i,\beta+j}(x) ,$$

where the B^r_{ij} *are independent from* x *and have the properties*:

(i) $B^r_{ij}(\alpha, 0) = 0$ *for* $j \geqq 1$;

(ii) $B^r_{r0}(\alpha, \beta) \neq 0$;

(iii) $\sum_{i,j=0}^{r} B^r_{ij}(\alpha, \beta) = (-1)^r r!$.

Received January 19, 1960.