TORSION ENDOMORPHIC IMAGES OF MIXED ABELIAN GROUPS

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In this paper we will answer Fuchs' PROBLEM 32 (a), and the corresponding part of his PROBLEM 33. (See [1], pg. 203.) The statements of these PROBLEMS are the following.

I. "Which are the torsion groups T that are endomorphic images of all groups containing them as maximal torsion subgroups?"

II. "Which are the torsion groups T such that a basic subgroup of T is an endomorphic image of any group G containing T as its maximal torsion subgroup?"

Actually, we will answer question II and the following question which is more general than I.

III. What groups H are endomorphic images of all groups G containing H such that G/H is torsion free?

The solutions will be effected by using some homological results of Harrison [2]. All groups considered here will be Abelian. The definitions and results stated in the remainder of this paragraph are due to Harrison, and may be found in [2]. A reduced group G is *cotorsion* if $\mathrm{Ext}(A, G) = 0$ for all torsion free groups A. If H is a reduced group, then $\mathrm{Ext}(Q/Z, H) = H'$ is cotorsion, where Q and Z denote the additive group of rationals and integers, respectively. Furthermore, H is a subgroup of H', (that is, there is a natural isomorphism of H into H') and H'/H is divisible torsion free. This implies, of course, that if T is a torsion reduced group, then T is the torsion subgroup of $T' = \mathrm{Ext}(Q/Z, T)$.

Now it is easy to see that *if G is a group such that* $\mathrm{Ext}(A, G) = 0$ *for all torsion free groups A, then any homomorphic image of G is the direct sum of a cotorsion group and a divisible group.* In fact, let H be a homomorphic image of G. This gives us an exact sequence

$$0 \to K \to G \to H \to 0$$

which yields the exact sequence

$$0 \to \mathrm{Hom}(A, K) \to \mathrm{Hom}(A, G) \to \mathrm{Hom}(A, H) \to \mathrm{Ext}(A, K) \to \mathrm{Ext}(A, G) \to \mathrm{Ext}(A, H) \to 0\ .$$

If A is any torsion free group, then $\mathrm{Ext}(A, G) = 0$, and so $\mathrm{Ext}(A, H) = 0$. Write $H = D \oplus L$, where D is the divisible part of H. Then L is reduced, and $0 = \mathrm{Ext}(A, D \oplus L) \cong \mathrm{Ext}(A, D) \oplus \mathrm{Ext}(A, L) = \mathrm{Ext}(A, L)$, so that L is cotorsion. Our assertion is proved.

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