CONSTRUCTION OF A CLASS OF MODULAR FUNCTIONS AND FORMS

MARVIN ISADORE KNOPP

1. Introduction. Let $G(j)$ be the principal congruence subgroup, of level j, of the modular group. In this paper we construct functions which are invariant under $G(j)$, for each integer $j \geqq 2$.

We begin by defining certain functions $\lambda_\nu(j; \tau)$ which, although not in general invariant under $G(j)$, do possess the transformation properties

(1.01) $\lambda_\nu(j; T\tau) = \lambda_\nu(j; \tau) + \text{constant}$, for all T in $G(j)$.

This is the content of the main theorem, Theorem (4.02). Once this result has been established it is a simple matter to construct invariants for $G(j)$ by forming certain linear combinations of the $\lambda_\nu(j; \tau)$. This is done in § 5.

These functions $\lambda_\nu(j; \tau)$ are defined as Fourier series which generalize the Fourier series expansion of $\lambda(\tau)$, given by Simons [6]. To derive the transformation equations (1.01), we proceed directly from the Fourier series, extending a method introduced by Rademacher [4], and since generalized by Lehner [2] and the author [1]. Although in [4] only the invariant $J(\tau)$ for the modular group is treated, the method of [4] has much wider applicability. Thus, in [2] it is used in the case of the modular group to overcome the usual convergence difficulties encountered in constructing forms of dimension -2 by means of Poincaré series, while in [1] it is used to construct forms of nonnegative even integral dimension (in which case we, of course, do not have the method of the Poincaré series) for the modular group and several other closely related groups.

We will indicate in section 6 how the method of this paper can be used to construct automorphic forms of all positive even integral dimensions for the groups $G(j)$. In a future publication these same methods will be applied to construct automorphic functions and forms for certain other congruence subgroups of the modular group and for congruence subgroups of several other groups.

I would like to thank the referee of this paper for his helpful remarks.

2. Several lemmas. In [4] the principal analytic tool is a rather delicate lemma in which the terms of a certain conditionally convergent double series are rearranged. Several variations of this lemma can be

Received January 20, 1960 The author is a National Science Fellow.