THE SUBGROUPS OF A DIVISIBLE GROUP G WHICH CAN BE REPRESENTED AS INTERSECTIONS OF DIVISIBLE SUBGROUPS OF G

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Introduction. In [1], page 70, L. Fuchs asks the following question: Which are those subgroups of a divisible group G that can be represented as intersections of divisible subgroups of G?

The main purpose of this paper is to give an answer to this question.

NOTATION.

N1: If H is a primary p-group, let $S(H)$ denote the subgroup of elements of H whose orders are 1 or p.

N2: If G is Abelian, let $T(G)$ be the torsion subgroup of G; let G_p denote the primary p-component of $T(G)$; and, in case G is divisible, let $F(G)$ denote a maximal torsion free subgroup of G.

N3: Let the symbol $\oplus$ denote a direct sum. Let the symbol $<$ denote "properly contained in." Let $\subset$ denote "contained in." Let $N\backslash M$ denote "the set of elements in N and not in M." Let $\cong$ denote "is isomorphic to." Let $\exists$ denote "there exists (exist)." Let $\ni$ denote "such that." Let $(N_a)_{a\in A}$ denote a family of sets N_a indexed by members of the set A. Finally if Q is a subset of a group, let $\{Q\}$ denote the subgroup of that group generated by the elements of Q.

N4: Let R denote the additive group of rationals. Let P denote the set of primes. Let the group $C(p^\infty)$ be the indecomposable divisible primary p-group.

N5: Let $C = C(2^\infty) \oplus C(3^\infty) \oplus C(5^\infty) \oplus \cdots$; and if $S \subset P$, let $C_S = \bigoplus_{p\in S} C(p^\infty)$.

N6: If G is a group, let $P(G)$ be the set of $p \in P$, such that $\exists x \in G$ with order $x = p$.

N7: Finally, we recall the following convenient and succinct classification of the subgroups of R [see Kurosh I, page 208]. Let $p_1, p_2, p_3, \cdots$ be the sequence of primes in natural order. A characteristic is a sequence $a = (a_1, a_2, a_3, \cdots)$, where $a_i =$ a non-negative integer or ∞. A type is a class of equivalent characteristics, two characteristic $a = (a_1, a_2, a_3, \cdots)$ and $b = (b_1, b_2, b_3, \cdots)$ being equivalent if and only if $\sum_{i=1}^{\infty} |a_i - b_i| < \infty$, where $\infty - \infty = 0$.

$A \subset R$ has type a if and only if it is isomorphic to the subgroup

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