A STRONG MAXIMUM PRINCIPLE FOR WEAKLY SUBPARABOLIC FUNCTIONS

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Introduction. It has been proved by E. Hopf [3], over thirty years ago, that solutions of second order elliptic equations satisfy the maximum principle. A similar principle, well known for solutions of the heat equation, has been, relatively recently, extended to second order parabolic equations by Nirenberg [5]. In various problems, such as in solving the Dirichlet problem by the methods of Poincaré and Perron, subsolutions have been introduced and the maximum principle has been extended to such functions. In the elliptic case (see [6]) the subsolutions used are continuous, whereas in the parabolic case, they may have certain discontinuities (see [2]). In the elliptic case, they are called L-subharmonic or subelliptic functions. Likewise, in the parabolic case, we call them L-subcaloric or subparabolic functions; L is the elliptic or the parabolic operator.

Recently, Walter Littman [4] has generalized the concept of L-subharmonic functions to include measurable integrable functions. This generalization is obtained by expressing the condition $Lu \geqq 0$ in an integrated form, namely, $\int uL^*vdx \geqq 0$ for any twice differentiable $v \geqq 0$ with compact support, L^* being the adjoint of L. He then established the maximum principle in the following sense: If an L-subharmonic function assumes its essential supremum at a point of continuity, then it is equal to a constant almost everywhere.

The purpose of this paper is to prove a similar result for measurable L-subcaloric functions. The general outline of the proof is similar to that of Littman's method. However, the crucial step in the proof is the construction of two kernal functions with certain required properties. Our construction is entirely different from that of Littman.

In § 1 we state some definitions and the results of the paper. In § 2 we prove Lemma 2. In § 3 we recall some properties of fundamental solutions. These are used in § 4 to prove Lemma 1. Lemmas 1, 2 immediately yield the maximum principle.

1. Statement of the results. Consider the differential operators

$$Lu \equiv \sum_{i,j=1}^{n} a_{ij}(x, t)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} a_i(x, t)\frac{\partial u}{\partial x_i} + a(x, t)u - \frac{\partial u}{\partial t}$$

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