RADIAL DISTRIBUTION AND DEFICIENCIES OF THE VALUES OF A MEROMORPHIC FUNCTION

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Introduction. Let $f(z)$ be a meromorphic function. Throughout this note we make the following conventions.

I. $f(0) = 1$; this simplifies the exposition without affecting the generality of the results.

II. We denote by

$$a_1, a_2, a_3, \cdots$$

the sequence of the zeros of $f(z)$ and by

$$b_1, b_2, b_3, \cdots$$

the sequence of its poles.

The moduli of the terms of these two sequences are taken to be nondecreasing and each zero or pole appears as often as indicated by its multiplicity.

III. The standard symbols of Nevanlinna's theory:

$$\log^+, \ m(r,f), \ \log M(r,f), \ n(r,f), \ N(r,f), \ T(r,f), \ \delta(\tau,f)$$

are used systematically; familiarity with their meaning is assumed.

We investigate here the following problem, a special case of which has already been mentioned by two of the authors [**1**; p. 295]:

To find sequences $\{a_\mu\}$, $\{b_\nu\}$ such that if $f(z)$ is a meromorphic function with zeros $\{a_\mu\}$ and poles $\{b_\nu\}$ (and no other zeros or poles), then

$$\delta(0,f) > 0 , \qquad \delta(\infty,f) > 0 . \tag{1}$$

The results of the present note show that a simple behavior of the arguments of the zeros and poles is almost sufficient to induce the inequalities (1). We prove

THEOREM 1. *Let $f(z)$ be a meromorphic function with positive zeros and negative poles.*

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