SOME GENERALIZATIONS OF METRIC SPACES

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1. Introduction. This paper consists of a study of certain classes of topological spaces (called M_1-, M_2-, and M_3-spaces) which include metric spaces and CW-complexes and are included in the class of all paracompact and perfectly normal spaces. It is shown, for example, that like the case in metric spaces, a subset of an M_2- (or M_3-) space is an M_2- (or M_3-) space; a countable product of M_i-spaces ($i = 1, 2, 3$) is again an M_i-space; and separable is equivalent to Lindelöf in an M_i-space. Moreover, unlike the case in metric spaces, the quotient space obtained by identifying the points of a closed subset of an M_2- (or M_3-) space is again an M_2- (or M_3-) space (for metric spaces such a quotient space need not be first countable). Also, we have $M_1 \rightarrow M_2 \rightarrow M_3$, but whether $M_3 \rightarrow M_2$ or $M_2 \rightarrow M_1$ is unknown.[1]

These classes of spaces are derived from generalizations of the following well-known characterization of metrizability in terms of specific properties of the base:

THEOREM 1.1. (Smirnov [14] or Nagata [12]). *A regular space is metrizable if and only if it has a σ-locally finite base.*

Recall that a σ-locally finite family is a union of countably many locally finite families. It is easily checked that a locally finite family $\boldsymbol{U}$ of sets has the property, called *closure preserving*, that for any

$$\boldsymbol{V} \subset \boldsymbol{U}, \quad (\cup \{V \in \boldsymbol{V}\})^- = \cup \{V^- : V \in \boldsymbol{V}\} .$$

This, then, suggests we consider spaces having a *σ-closure preserving base* (that is, a base which is the union of countably many closure preserving families).

DEFINITION 1.1. An M_1-space is a regular space having a σ-closure preserving base.

Although conceptually simple, M_1-spaces prove unsatisfactory in some respects, so we weaken the condition of having a σ-closure preserving base. We begin by calling a collection $\boldsymbol{B}$ of (not necessarily open!) subsets of X a *quasi-base* if, whenever $x \in X$ and U is a neighborhood of

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[1] Nearly all topological terminology appearing in this paper is consistent with that used in Kelley [4]. Exceptions are that our regular, and normal spaces are assumed to be T_1-spaces.