DISCONJUGACY OF A SELF-ADJOINT DIFFERENTIAL EQUATION OF THE FOURTH ORDER[1]

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Introduction. In a recent paper [10] W. Leighton and Z. Nehari investigated oscillation properties of solutions of self-adjoint differential equations of the fourth order

$$(r(x)y'')'' + (q(x)y')' + p(x)y = 0$$

with particular attention to the cases where the middle term is missing, $r(x) > 0$ and $p(x)$ does not change sign. In the present paper one of these particular cases

$$(r(x)y'')'' - p(x)y = 0 \tag{1}$$

($r(x)$ and $p(x)$ positive and continuous on $[a, \infty)$) will be pursued further with the object of paralleling the known theory of second order equation

$$(r(x)y')' + p(x)y = 0 \tag{2}$$

with positive and continuous coefficients (e.g., see [2] and [12]). With only occasional minor modifications the terminology of [10], together with the fundamental properties of (1) established there, will be assumed throughout this paper. One point of departure is the distinction between "disconjugacy" and "non-oscillation" as the author has used them previously [2] for equation (2) in discussions which will be extended here to the fourth-order equation (1). It will be said that equation (1) is

(i) *disconjugate* if no nontrivial solution has more than 3 zeros on $[a, \infty)$ and, hence, no conjugate pairs exist on $[a, \infty)$ in the sense of Leighton and Nehari [10],

(ii) *oscillatory* if there is a nontrivial solution with infinitely many zeros on $[a, \infty)$.

(iii) *nonoscillatory* if every nontrivial solution has at most a finite number of zeros on $[a, \infty)$.

Recently, W. J. Coles [5] has developed Wirtinger-type inequalities in relation to the higher order equation

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