GENERALIZED TWISTED FIELDS

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1. Introduction. Consider a finite field $\mathfrak{K}$. If V is any automorphism of $\mathfrak{K}$ we define $\mathfrak{K}_V$ to be the *fixed field* of K under V. Let S and T be any automorphism of $\mathfrak{K}$ and define F to be the fixed field

$$(1) \qquad \mathfrak{F} = \mathfrak{F}_q = (\mathfrak{K}_S)_T = (\mathfrak{K}_T)_S ,$$

under both S and T. Then $\mathfrak{F}$ is the field of $q = p^\alpha$ elements, where p is the characteristic of $\mathfrak{K}$, and $\mathfrak{K}$ is a field of degree n over $\mathfrak{F}$. We shall assume that

$$(2) \qquad n > 2 , \qquad q > 2 .$$

Then the period of a primitive element of $\mathfrak{K}$ is $q^n - 1$ and there always exist elements c in $\mathfrak{K}$ such that $c \neq k^{q-1}$ for any element k of $\mathfrak{K}$. Indeed we could always select c to be a primitive element of $\mathfrak{K}$.

Define a product (x, y) on the additive abelian group $\mathfrak{K}$, in terms of the product xy of the field $\mathfrak{K}$, by

$$(3) \qquad (x, y) = xA_y = yB_x = xy - c(xT)(yS) ,$$

for c in $\mathfrak{K}$. Then

$$(4) \qquad A_y = R_y - TR_{c(yS)} , \qquad B_x = R_x - SR_{c(xT)} ,$$

where the transformation $R_y = R[y]$ is defined for all y in $\mathfrak{K}$ *by the product* $xy = xR_y$ of $\mathfrak{K}$. Then the condition that $(x, y) \neq 0$ for all $xy \neq 0$ is equivalent to the property that

$$(5) \qquad c \neq \frac{x}{xT}\frac{y}{yS} ,$$

for any nonzero x and y of $\mathfrak{K}$. But the definition of a generating automorphism U of $\mathfrak{K}$ over $\mathfrak{F}$ by $xU = x^q$ implies that

$$(6) \qquad S = U^\beta , \qquad T = U^\gamma .$$

We shall assume that $S \neq I$, $T \neq I$, so that

$$(7) \qquad 0 < \beta < n , \qquad 0 < \gamma < n .$$

Then $xy[(xS)(yT)]^{-1} = z^{q-1}$, where

$$(8) \qquad 1 - q^\beta = (q-1)^\delta,\ 1 - q^\gamma = (q-1)^\varepsilon,\ z = x^\delta y^\varepsilon .$$

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