ON SOME CLASSES OF SCALAR-PRODUCT ALGEBRAS

J. F. SMITH AND P. P. SAWOROTNOW

1. Introduction. In the second author's previous paper [10] a *two-sided H^*-algebra* was defined as a complex Banach algebra which is a Hilbert space, and which possesses two conjugate-linear bounded mappings $x \to x^l$ and $x \to x^r$ with the property that for any x, y, and z in the algebra, $(xy, z) = (y, x^l z) = (x, zy^r)$. This concept generalized the original definition of an H^*-algebra given by Ambrose [1]. It may readily be seen that in a two-sided H^*-algebra the orthogonal complement of a right (left) ideal is again an ideal of the same kind. It is shown in [10], moreover, that this "right (left) complementation" property is sufficient to characterize a two-sided H^*-algebra A without the assumption of the mappings $x \to x^l$ and $x \to x^r$, provided that A is an annihilator algebra in the sense of Bonsall and Goldie [5], that is, provided that every proper right (left) ideal of A has a nonzero left (right) annihilator.

The present paper will carry out a study that bears somewhat the same relationship to the Hilbert algebras of Nakano [7] as does the above-mentioned investigation in [10] to Ambrose's H^*-algebras. The results here, however, will be more restricted, since Hilbert algebras (and the systems similar to them: see the papers of Ambrose [2], Segal [12], Godement [6], and Pallu de la Barrière [8]) are much more general and less manageable than H^*-algebras. In particular, we shall have neither joint continuity of multiplication in the algebra nor completeness of the metric space formed by its elements under the scalar-product norm. These strong properties are lacking for Hilbert algebras in general; in addition, however, we shall replace the standard assumption of the existence of a conjugate-linear isometry and the adjoint character of this mapping by the requirement that in our algebras the orthogonal complement of a right ideal shall be a right ideal. To compensate somewhat for this loss, our considerations will be restricted to a class of algebras that may be described as symmetric, maximal, and topologically semi-simple. We shall define these terms in the following section, in which we discuss some matters corresponding for our case to the theory of regular ideals fundamental in the study of Banach algebras.

2. Preliminary theory. We shall deal with algebras possessing some of the properties of Hilbert algebras, apart from the $*$-mapping.

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