SOME GENERALIZATIONS OF DOEBLIN'S DECOMPOSITION

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Introduction. We consider a Markov chain $\{X_i\}$ $i = 0, 1, \cdots$ with stationary transition probabilities $P^k(t, E)$ defined on a measure space (Ω, Σ). All sets discussed in the following will be Σ-sets. A set N is called null if $P^1(t, N) = P(t, N) = 0$ for all $t \in \Omega$, and a set S is called invariant if $P(t, S) = 1$ for $t \in S - N$ where N is a null set. $\mathscr{I}_p$ will denote the σ-field determined by the invariant sets given the transition probability $P(t, E)$. A set S is indecomposable if it does not contain two disjoint non-null invariant subsets. The concept of a strictly separable σ-field will be employed, together with the fact that such a σ-field is atomic. S^c is the complement of the set S.

This paper considers several conditions under which we have a general decomposition $\Omega = F + \sum_\alpha A_\alpha$ where F is a transient state and the A_α are ergodic, indecomposable state, i. e., defining

$$P_1(t, E) = \lim_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} P^k(t, E) ,$$

then $P_1(t, \sum_\alpha A_\alpha) = 1$ for all $t \in \Omega$, $P(t, A_\alpha) = 1$ for $t \in A_\alpha$, and the A_α are minimal, up to an equivalence. This work may be considered as a further step in Doob's discussion in [3] on generalizing Doeblin's classical results. Our results are sometimes generalizations of Doob's work and other times give slightly stronger conclusions, but replace Doob's assumption of an a priori stationary measure for the process by general conditions in terms of measures.

Theorem 1 is due to Blackwell and is the basis for Theorem 2, the decomposition theorem, which is proved under the assumption of the existence of the Cesàro limit $P_1(t, E)$ for all $t \in \Omega$, $E \in \Sigma$. Theorem 3 gives Doeblin type conditions in terms of measures implying the existence of $P_1(t, E)$. Theorem 4 discusses the special case of a priori knowledge of a σ-finite stationary measure for the process. Finally, Theorem 5 gives a countable decomposition when the Cesàro limit is absolutely continuous with respect to a σ-finite measure.

THEOREM 1. (Blackwell). *Let $P(t, E)$ be an idempotent Markov*

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The author takes this opportunity to express his thanks to the referee for several suggested modifications, including the correction of a mistake in the original proof of Theorem 2. The revised proof is presented here.