AUTOMORPHISMS OF MONOMIAL GROUPS

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If H be an arbitrary group and S a set, then one obtains a monomial group after the manner described in [2]. Ore has in [2] determined the automorphisms of the monomial group when the set S has finite order. Here we obtain all automorphisms of a large class of monomial groups when the order of the set S is infinite. A monomial substitution over H is a linear transformation mapping each element x of S in a one-to-one manner onto some element of S multiplied formally by an element h of H. The element h is termed a factor of the substitution. A substitution all of whose factors are the identity e of H is called a permutation, while a substitution which maps each element of S into itself multiplied by an element of H is called a multiplication. A multiplication all of whose factors are equal is termed a scalar. The monomial substitutions restricted by the definitions of C and D as given below are elements of the monomial group denoted by $\Sigma(H;\ B, C, D)$, where the symbols in the name are to be interpreted as follows, H the given group, B the order of the given set S, C a cardinal number such that the number of non-identity factors of any substitution is less than C, D a cardinal number such that the number of elements of S being mapped into elements of S distinct from themselves by a substitution is less than D. $S(B, D)$ will denote the subgroup consisting of all permutations, while $V(B, C)$ will denote the subgroup consisting of all multiplications. Any substitution u may be written as the product of a multiplication v and a permutation s. Hence we may write $\Sigma(H;\ B, C, D) = V(B, C) \cup S(B, D)$, where $\cup$ here and throughout will mean group generated by the set.

The main result of this paper is to determine all automorphisms of the monomial group $\Sigma(H;\ B, d, C)$, $d \leqq C < B^+$, where B^+ is the successor of B, $d = \aleph_0$ and to determine the automorhism group of this group.

LEMMA 1. *The basis group* $V(B, d)$ *is a characteristic subgroup of* $\Sigma(H\colon\ B, d, C)$, $d \leqq C \leqq B^+$.

Crouch has shown in [1] that $V(B, d)$ is a characteristic subgroup of $\Sigma(H;\ B, d, d)$. It is easy to show that if N is a subgroup of $V(B, d)$, then N is normal in $\Sigma(H;\ B, d, d)$ if and only if N is normal in $\Sigma(H;\ B, C, D)$. With this result Lemma 1 is an easy generalization of the result of Crouch.

LEMMA 2. *If* T *is an endomorphism of* $V(B, d)$, *then there exist*

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